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CLIQUES PROPERTIES FROM DIAGONAL LATIN SQUARES OF SMALL ORDER

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What is Latin squares?

$$A = \|a_{ij}\|$$

$$i,\ j = \overline{1,\ N}$$

$$N = |S|$$

$$S = \{0, 1, 2, \ldots, N-1\}$$

$$\forall i,\ j, k = \overline{1, N},\ j \neq k : (a_{ij} \neq a_{ik}) \wedge (a_{ji} \neq a_{ki})$$

$$\forall i,\ j = \overline{1, N}, i \neq j : (a_{ii} \neq a_{jj}) \wedge (a_{N-i+1,\ N-i+1} \neq a_{N-j+1,\ N-j+1})$$

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 9 & 4 & 3 & 6 & 7 & 5 & 0 & 8 \\ 2 & 9 & 3 & 1 & 7 & 0 & 5 & 8 & 4 & 6 \\ 3 & 4 & 1 & 2 & 8 & 7 & 9 & 6 & 5 & 0 \\ 4 & 3 & 5 & 9 & 2 & 1 & 8 & 0 & 6 & 7 \\ 5 & 6 & 4 & 8 & 1 & 2 & 0 & 9 & 7 & 3 \\ 6 & 5 & 8 & 7 & 0 & 3 & 2 & 1 & 9 & 4 \\ 7 & 8 & 6 & 0 & 9 & 4 & 1 & 2 & 3 & 5 \\ 8 & 7 & 0 & 5 & 6 & 9 & 3 & 4 & 1 & 2 \\ 9 & 0 & 7 & 6 & 5 & 8 & 4 & 3 & 2 & 1 \end{pmatrix}$$

Normalized LS of order 10

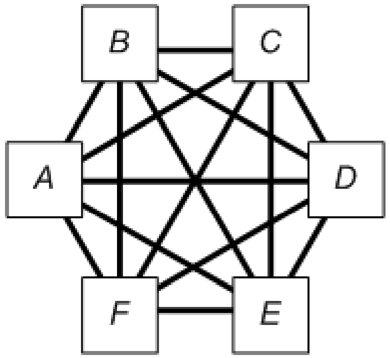
$$N! \times (N-1)!$$

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 2 & 4 & 9 & 0 & 6 & 5 & 1 & 3 & 8 \\ 8 & 3 & 6 & 7 & 5 & 9 & 0 & 2 & 4 & 1 \\ 2 & 6 & 8 & 5 & 1 & 7 & 4 & 0 & 9 & 3 \\ 5 & 8 & 9 & 1 & 7 & 0 & 3 & 4 & 6 & 2 \\ 9 & 4 & 1 & 2 & 8 & 3 & 7 & 6 & 0 & 5 \\ 4 & 7 & 5 & 6 & 9 & 1 & 8 & 3 & 2 & 0 \\ 3 & 0 & 7 & 8 & 2 & 4 & 1 & 9 & 5 & 6 \\ 6 & 5 & 0 & 4 & 3 & 2 & 9 & 8 & 1 & 7 \\ 1 & 9 & 3 & 0 & 6 & 8 & 2 & 5 & 7 & 4 \end{pmatrix}$$

Normalized DLS of order 10

$$(N-1)!$$

Why is this interesting?



Applied problems:

- experiment planning
- cryptography
- error correcting codes
- scheduling
- algebra, combinatorics, statistics, ...

Mathematical problems:

- **existence of a triple of MOLS / MODLS of order 10 (or larger clique)**
- increasing world record of orthogonality characteristic for pseudo triple of MOLS (291/300) or MODLS (274/300)
- generating functions
- asymptotic behavior of combinatorial characteristics based on DLSs (OEIS)
- number theory (relations between different fields of knowledge)
- magic squares
- Sudoku (LS of order 9 with additional constraints)

Brief history and approaches

- Euler — triple of MOLS of order 10 does not exist (**disproved**);
- Parker et al. (1960) — **first pair of MOLS** of order 10 using transversals;
- Brown et al. (1995) — horyzontally symmetric DLS, **1:4**;
- Zaikin et al. (2015—2016) — SAT approach (system of Boolean equations), ~100 CFs of ODLS, **onces**;
- Vatutin et al. (from 2017) — **RS+LBF generator -> transversals + DLX**, ~1M CFs of ODLS, **twices**, **1:3**;
- Vatutin et al. (2017) — **plane symmetry generator -> transversals + DLX**, ~200k CFs of ODLS, **1:6**, **1:8**, **rhombus-4**, **line-4**, **line-5**, **loop-4**, **fish**;
- Vatutin et al. (from 2017) — **central symmetry, partial central and plane symmetries, generalized symmetries, neighborhoods of generalized symmetris generator -> canonizer -> postprocessor**, ~7M CFs of ODLS, **1:5**, **1:7**, **1:10**, **rhombus-3**, **cross**, **flyer**, **tree-1**, **Venus**, **Daedalus-8**, **Daedalus-10**, **robot**, **stingray**;
- Vatutin et al. (from 2019) — **none transversal search for ODLS.**

Classical search for pairs of ODLS of order 10

L. Euler expected that for N=10 ODLS doesn't exist
First pair — Parker et al., 1960

0	1	2	3	4	5	6	7	8	9
1	2	0	4	3	7	9	8	5	6
7	3	5	9	0	4	8	6	2	1
3	5	6	8	9	0	4	1	7	2
4	9	7	2	6	8	1	5	0	3
5	8	4	6	7	1	3	2	9	0
8	4	9	1	2	3	7	0	6	5
6	7	3	0	1	2	5	9	4	8
9	0	1	5	8	6	2	4	3	7
2	6	8	7	5	9	0	3	1	4

0	1	2	3	4	5	6	7	8	9
7	5	1	9	2	8	0	4	6	3
1	0	3	4	6	7	5	2	9	8
9	8	4	7	5	2	1	0	3	6
6	7	9	0	8	3	2	1	5	4
4	6	5	1	0	9	8	3	2	7
2	3	8	5	1	6	4	9	7	0
5	2	7	8	3	4	9	6	0	1
3	4	6	2	9	0	7	8	1	5
8	9	0	6	7	1	3	5	4	2

SAT@Home, 04.2015

0	1	2	3	4	5	6	7	8	9
4	9	0	8	5	6	3	1	2	7
2	5	7	9	6	4	0	8	1	3
9	0	4	6	8	7	1	5	3	2
6	7	5	2	1	3	8	0	9	4
1	8	3	5	7	2	9	6	4	0
7	3	1	0	9	8	4	2	6	5
8	2	6	4	0	9	5	3	7	1
3	4	8	1	2	0	7	9	5	6
5	6	9	7	3	1	2	4	0	8

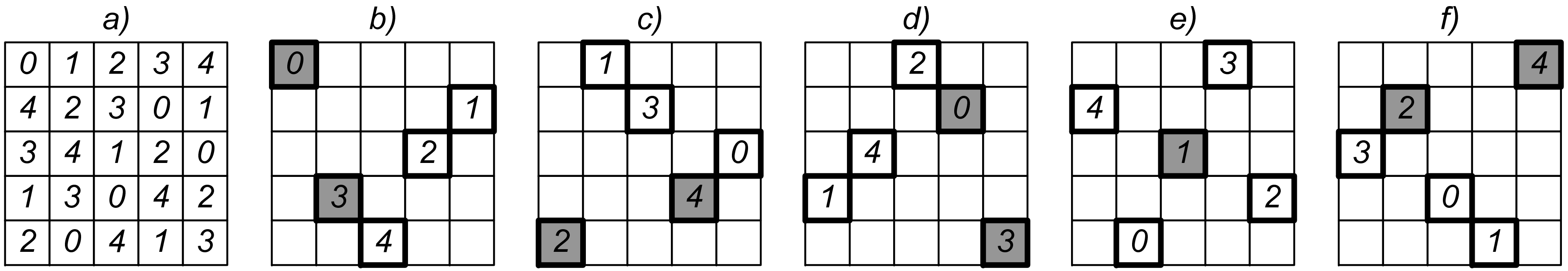
0	1	2	3	4	5	6	7	8	9
6	5	9	7	0	8	2	3	1	4
4	7	1	2	3	9	8	0	6	5
1	2	0	4	5	3	7	6	9	8
2	6	8	0	9	4	1	5	3	7
8	4	6	9	2	7	0	1	5	3
5	0	4	6	8	2	3	9	7	1
9	3	5	1	7	6	4	8	0	2
7	8	3	5	6	1	9	4	2	0
3	9	7	8	1	0	5	2	4	6

Gerasim@Home, 04.2017

Very rare combinatorial objects:
~30 millions DLS of order 10
has only **1 pair of ODLS**!

Searching for ODLS: approaches

- Brute Force + backtracking + clippings + ordering + ... (very long)
- SAT (some tens of hours, long)
- filling by pairs of elements [a_{ij}, b_{ij}] (long)
- using transversals (fast) – **200 – 800 DLS/s** for different algorithms!
- using transversals with canonizer (**~8000 DLS/s** effective pace)



$T^{(d)}_1=\{a_{11}, a_{25}, a_{34}, a_{42}, a_{53}\}$

$T^{(d)}_2=\{a_{12}, a_{23}, a_{35}, a_{43}, a_{51}\}$

$T^{(d)}_3=\{a_{13}, a_{24}, a_{32}, a_{41}, a_{55}\}$

$T^{(d)}_4=\{a_{14}, a_{21}, a_{33}, a_{45}, a_{52}\}$

$T^{(d)}_5=\{a_{15}, a_{22}, a_{31}, a_{43}, a_{54}\}$

Neighborhoods of generalized symmetries (01.11.2019)

Обработка области (Р — Разведка (10k), С — случайный перебор окрестности (100k+), П — полный перебор)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
1	-	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	РП	Р	Р	Р	Р	Р	Р	Р	Р	Р	РП	РП
2		Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р					Р								РП	Р				Р					Р	РП
3			Р	Р	Р			Р				Р				Р							Р								Р	Р				Р		Р			Р	
4				Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р				Р			Р	Р	Р			РП	Р				Р			Р		Р	РП
5					Р			Р				Р				Р			Р				Р		Р				Р		Р	Р		Р	Р	Р	Р		Р		Р	
6								Р				Р				Р			Р				Р		Р				Р		Р	Р							Р		Р	
7							РП	Р				Р				Р				Р		Р			Р				Р		Р	Р							Р		РП	РП
8								РП	Р	Р	Р	Р	Р	Р	Р	Р	Р	Р			Р		Р		Р				Р		РП	Р		Р	Р	Р	Р		Р		Р	РП
9												Р	Р	Р	Р	Р				Р		Р			Р				Р		Р								Р			
10										РП		Р				Р			Р				Р		Р				Р		Р			РП				Р				
11											РП	Р				Р			Р				Р		Р				Р		Р			РП								
12												Р	Р	Р	Р	Р	Р		Р				Р		Р				Р		Р											
13													Р	Р	Р	Р			Р				Р			Р	Р	Р			Р											РП
14																Р																										
15															РП	Р																										
16																РП	Р						Р								РП											РП
17																															Р				Р	РП		Р	Р			
18																																		РП			Р			Р		
19																			РП												Р			РП								
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38																																										
39																																										
40																																										
41																																									РП	РП
42																																										РП

- 268 from 903 neighborhoods processed (29,7%)

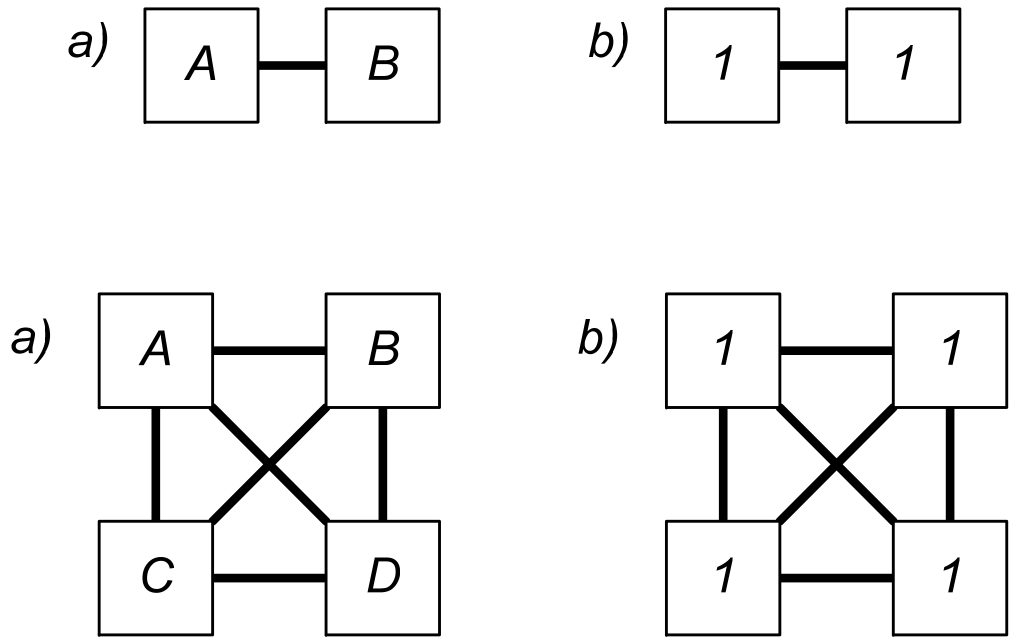
Properties of cliques within combinatorial structures of small order (N=4)

- clique-2, **<u>1 CF</u>**

Properties of cliques within combinatorial structures of small order (N=5)

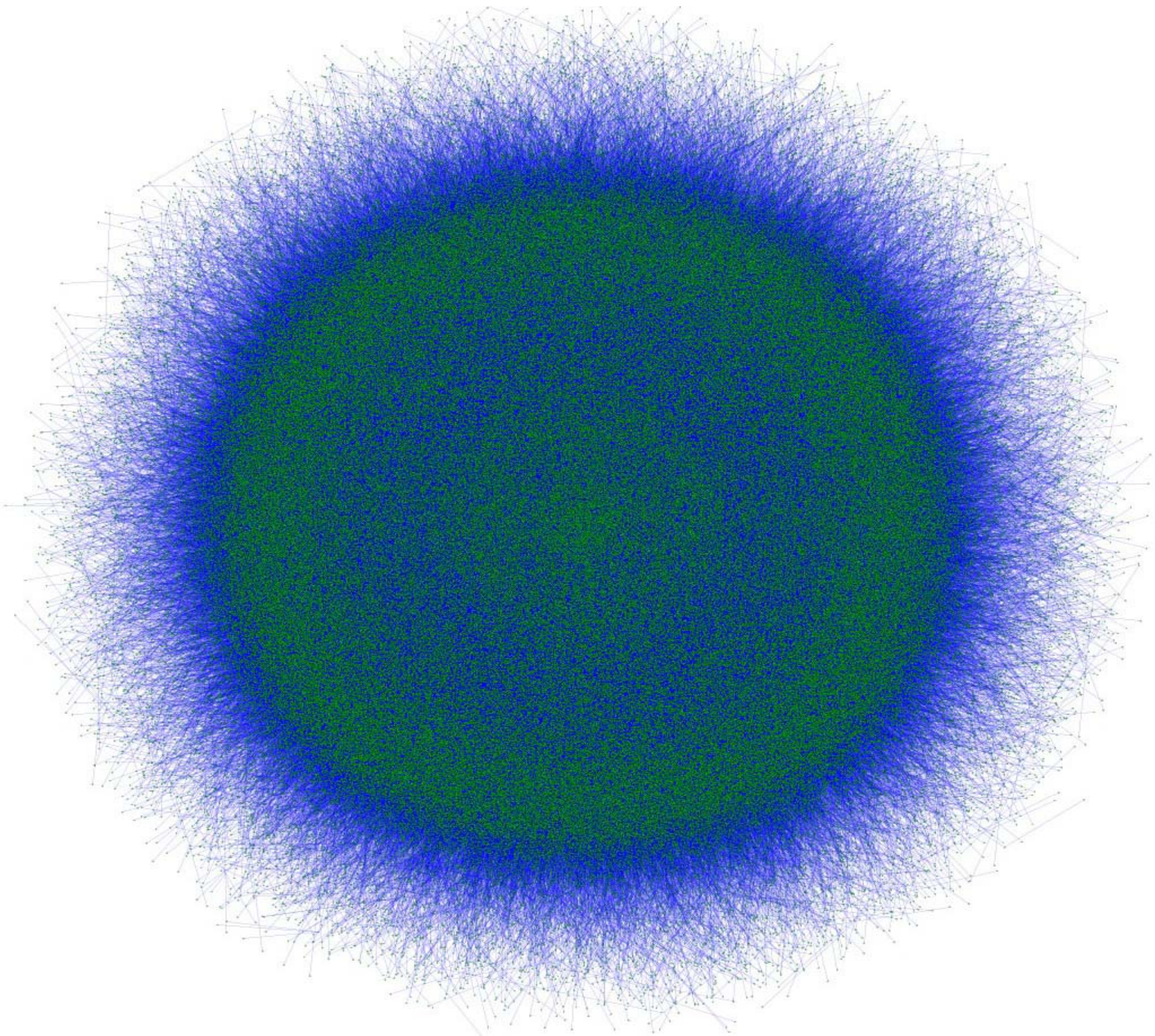
- clique-2, **1 CF**

Properties of cliques within combinatorial structures of small order (N=7)



- clique-2, **1 CF**
- clique-4, **1 CF**

Properties of cliques within combinatorial structures of small order (N=8)



- N824HUGE structure (348000 DLSs, 657 CFs), includes clique-6, **1 CF**

Properties of cliques within combinatorial structures of small order (N=9)

197 different combinatorial structures (some of them with cliques):

- **24N54M4C** — clique-3, 1 CF;
- **120N480M5C** — clique-4, 1 CF;
- **24N60M4C** — clique-4, 1 CF;

- **32N86M13C** — clique-3, **3 CFs**;
- **48N126M6C** — clique-6, **2 CFs**.

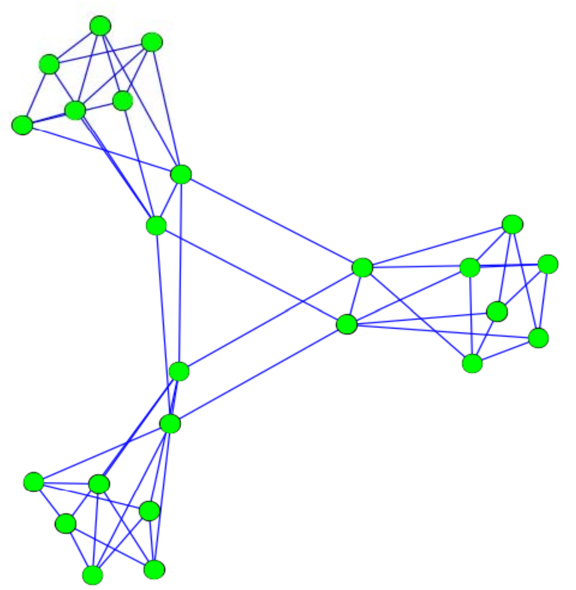


Figure 148. Graph of ODLS from workunit R9_000463421/02.
2017.12.15. Rake Search project. [B@P] Daniel (BOINC@Poland) and LCB001 (Hardware Canucks)

http://evatutin.narod.ru/evatutin_ls_all_structs_n1to8_eng.pdf
http://evatutin.narod.ru/evatutin_ls_all_structs_n9_eng.pdf
http://evatutin.narod.ru/evatutin_ls_all_structs_eng.pdf

- based on RakeSearch project results (https://rake.boincfast.ru/rakesearch/)

Properties of cliques within combinatorial structures of order N=10

More then clique-2 subgraphs don't known...

Searching for ODLS from same main class?

0	1	2	3	4	5	6	7	8	9
5	9	6	4	8	1	3	0	2	7
9	0	1	8	6	2	7	4	5	3
4	6	5	2	0	7	8	3	9	1
2	4	9	7	3	6	1	8	0	5
3	7	8	9	5	4	0	2	1	6
7	8	3	0	2	9	5	1	6	4
8	5	7	1	9	0	4	6	3	2
6	3	4	5	1	8	2	9	7	0
1	2	0	6	7	3	9	5	4	8

SODLS

- Bennett F.E., Beiliang Du, Hantao Zhang. Existence of self-orthogonal diagonal Latin squares with a missing subsquare // Discrete Mathematics. Vol. 261. 2003. pp. 69-86.

SODLS can be extended for ESODLS

ESODLS (Ed's SODLS) — MODLS from DLSs within same main class

OEIS sequences (SODLS, H. White):

- A287761 — **1, 0, 0, 2, 4, 0, 64, 1152, 224832;**
- A287762 — **1, 0, 0, 48, 480, 0, 322560, 46448640, 81587036160.**

OEIS sequences (ESODLS, new):

- A309210 — **1, 0, 0, 1, 1, 0, 5, 23;**
- A309598 — **1, 0, 0, 2, 4, 0, 256, 4608;**
- A309599 — **1, 0, 0, 48, 480, 0, 1290240, 185794560.**

This site is supported by donations to The OEIS Foundation.

0 1 3 6 2 7 13 20 12 21 11 22 10 23 OEIS THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES ®

founded in 1964 by N. J. A. Sloane

Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

A309598 Number of extended self-orthogonal diagonal Latin squares of order n with ordered first string. 0

1, 0, 0, 2, 4, 0, 256, 4608 (list; graph; refs; listen; history; text; internal format)

OFFSET 1,4

COMMENTS A self-orthogonal diagonal Latin square (SODLS) is a diagonal Latin square orthogonal to its transpose. An extended self-orthogonal diagonal Latin square (ESODLS) is a diagonal Latin square that has an orthogonal diagonal Latin square from the same main class. SODLS is a special case of ESODLS.

LINKS Table of n, a(n) for n=1..8.
E. I. Vatutin, Discussion about properties of diagonal Latin squares (in Russian)
Index entries for sequences related to Latin squares and rectangles

EXAMPLE The diagonal Latin square

```
0 1 2 3 4 5 6 7 8 9
1 2 0 4 5 7 9 8 6 3
5 0 1 6 3 9 8 2 4 7
9 3 5 8 2 1 7 4 0 6
4 6 3 5 7 8 0 9 2 1
8 4 6 9 1 3 2 5 7 0
7 8 9 0 6 4 5 1 3 2
2 9 4 7 8 0 3 6 1 5
6 5 7 1 0 2 4 3 9 8
3 7 8 2 9 6 1 0 5 4
```

has orthogonal diagonal Latin square

```
0 1 2 3 4 5 6 7 8 9
3 5 9 8 6 2 0 1 4 7
4 3 8 7 2 1 9 0 5 6
6 9 3 4 8 0 1 2 7 5
7 2 0 1 9 3 5 8 6 4
2 0 1 5 7 6 4 9 3 8
8 6 4 2 0 9 7 5 1 3
1 7 6 0 5 4 8 3 9 2
9 8 5 6 1 7 3 4 2 0
5 4 7 9 3 8 2 6 0 1
```

from the same main class.

CROSSREFS Cf. A287761.
Sequence in context: A287761 A009512 A317411 * A305570 A287651 A163259
Adjacent sequences: A309595 A309596 A309597 * A309599 A309600 A309601

KEYWORD nonn,more

AUTHOR Eduard I. Vatutin, Aug 09 2019

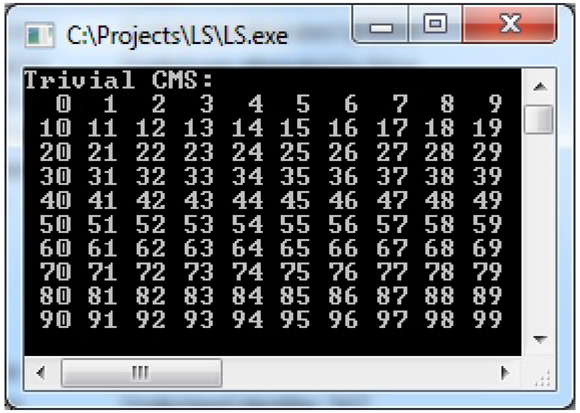
STATUS approved

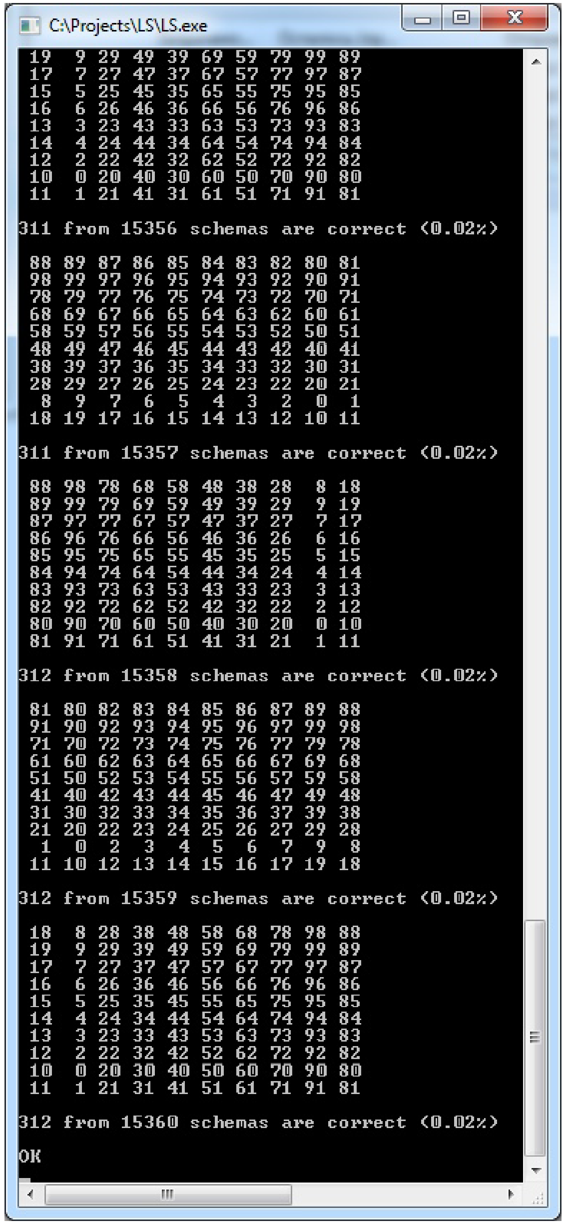
How we can find ESODLS? CMS-based search…

Bijective mapping for N^2 cells of square with some special properties

SODLS — one of them…

M-transformations





Different properties of CMS: example 1

CMS[0] = 0
CMS[1] = 1
CMS[2] = 2
CMS[3] = 3
CMS[4] = 4
CMS[5] = 5
CMS[6] = 6
CMS[7] = 7
CMS[8] = 8
CMS[9] = 9
CMS[10] = 30
CMS[11] = 95
CMS[12] = 32
CMS[13] = 35
CMS[14] = 48
CMS[15] = 92
CMS[16] = 93
CMS[17] = 78
CMS[18] = 65
CMS[19] = 45
CMS[20] = 31
CMS[21] = 51
CMS[22] = 83
CMS[23] = 80
CMS[24] = 47
CMS[25] = 39
CMS[26] = 72
CMS[27] = 89
CMS[28] = 40
CMS[29] = 82
CMS[30] = 10
CMS[31] = 20
CMS[32] = 12
CMS[33] = 34
CMS[34] = 33
CMS[35] = 13
CMS[36] = 59
CMS[37] = 60
CMS[38] = 50
CMS[39] = 25
CMS[40] = 28
CMS[41] = 90
CMS[42] = 53
CMS[43] = 67
CMS[44] = 94
CMS[45] = 19
CMS[46] = 64
CMS[47] = 24
CMS[48] = 14
CMS[49] = 62
CMS[50] = 38
CMS[51] = 21
CMS[52] = 70
CMS[53] = 42
CMS[54] = 88
CMS[55] = 99
CMS[56] = 76
CMS[57] = 77
CMS[58] = 74
CMS[59] = 36
CMS[60] = 37
CMS[61] = 91
CMS[62] = 49
CMS[63] = 73
CMS[64] = 46
CMS[65] = 18
CMS[66] = 84
CMS[67] = 43
CMS[68] = 69
CMS[69] = 68
CMS[70] = 52
CMS[71] = 81
CMS[72] = 26
CMS[73] = 63
CMS[74] = 58
CMS[75] = 79
CMS[76] = 56
CMS[77] = 57
CMS[78] = 17
CMS[79] = 75
CMS[80] = 23
CMS[81] = 71
CMS[82] = 29
CMS[83] = 22
CMS[84] = 66
CMS[85] = 96
CMS[86] = 87
CMS[87] = 86
CMS[88] = 54
CMS[89] = 27
CMS[90] = 41
CMS[91] = 61
CMS[92] = 15
CMS[93] = 16
CMS[94] = 44
CMS[95] = 11
CMS[96] = 85
CMS[97] = 98
CMS[98] = 97
CMS[99] = 55

0	1	2	3	4	5	6	7	8	9
1	2	0	4	3	6	5	9	7	8
2	0	1	5	6	3	4	8	9	7
3	4	5	7	9	8	1	6	2	0
5	7	4	9	8	0	2	1	6	3
6	9	8	1	2	4	7	3	0	5
8	3	7	6	0	1	9	2	5	4
9	6	3	0	7	2	8	5	4	1
7	5	6	8	1	9	0	4	3	2
4	8	9	2	5	7	3	0	1	6

1	3	4	5	6	7	0	8	9	2
5	8	7	9	0	2	4	6	3	1
6	2	9	8	3	1	5	4	7	0
3	4	1	2	8	6	7	9	0	5
2	6	3	4	7	9	1	0	5	8
4	1	2	6	5	0	9	7	8	3
0	9	5	1	4	8	3	2	6	7
9	7	6	0	1	3	8	5	2	4
7	0	8	3	2	5	6	1	4	9
8	5	0	7	9	4	2	3	1	6

82 hours per ODLS pair

(**~10 times slower**)

Different properties of CMS: example 2

CMS[0] = 0
CMS[1] = 44
CMS[2] = 77
CMS[3] = 38
CMS[4] = 97
CMS[5] = 47
CMS[6] = 16
CMS[7] = 43
CMS[8] = 84
CMS[9] = 9
CMS[10] = 10
CMS[11] = 23
CMS[12] = 36
CMS[13] = 26
CMS[14] = 31
CMS[15] = 83
CMS[16] = 6
CMS[17] = 98
CMS[18] = 79
CMS[19] = 80
CMS[20] = 41
CMS[21] = 21
CMS[22] = 39
CMS[23] = 11
CMS[24] = 51
CMS[25] = 74
CMS[26] = 13
CMS[27] = 75
CMS[28] = 49
CMS[29] = 95
CMS[30] = 94
CMS[31] = 14
CMS[32] = 76
CMS[33] = 33
CMS[34] = 42
CMS[35] = 45
CMS[36] = 12
CMS[37] = 91
CMS[38] = 3
CMS[39] = 22
CMS[40] = 60
CMS[41] = 20
CMS[42] = 34
CMS[43] = 7
CMS[44] = 1
CMS[45] = 35
CMS[46] = 58
CMS[47] = 5
CMS[48] = 52
CMS[49] = 28
CMS[50] = 50
CMS[51] = 24
CMS[52] = 48
CMS[53] = 54
CMS[54] = 53
CMS[55] = 87
CMS[56] = 93
CMS[57] = 64
CMS[58] = 46
CMS[59] = 73
CMS[60] = 40
CMS[61] = 67
CMS[62] = 70
CMS[63] = 86
CMS[64] = 57
CMS[65] = 96
CMS[66] = 99
CMS[67] = 61
CMS[68] = 78
CMS[69] = 71
CMS[70] = 62
CMS[71] = 69
CMS[72] = 72
CMS[73] = 59
CMS[74] = 25
CMS[75] = 27
CMS[76] = 32
CMS[77] = 2
CMS[78] = 68
CMS[79] = 18
CMS[80] = 19
CMS[81] = 85
CMS[82] = 82
CMS[83] = 15
CMS[84] = 8
CMS[85] = 81
CMS[86] = 63
CMS[87] = 55
CMS[88] = 92
CMS[89] = 89
CMS[90] = 90
CMS[91] = 37
CMS[92] = 88
CMS[93] = 56
CMS[94] = 30
CMS[95] = 29
CMS[96] = 65
CMS[97] = 4
CMS[98] = 17
CMS[99] = 66

0	6	5	7	4	8	9	3	2	1
2	1	8	6	5	9	4	0	7	3
4	3	9	8	7	1	5	2	0	6
8	7	3	4	9	5	0	1	6	2
1	8	0	9	2	3	6	7	4	5
5	9	2	3	8	6	7	4	1	0
7	0	4	5	1	2	3	6	8	9
3	5	6	2	0	7	1	8	9	4
6	4	7	1	3	0	2	9	5	8
9	2	1	0	6	4	8	5	3	7

0	5	6	7	2	3	4	1	8	9
5	6	0	2	3	9	1	8	4	7
6	8	5	9	1	0	7	3	2	4
7	2	9	4	0	8	6	5	3	1
3	4	1	8	7	2	9	6	5	0
2	3	4	6	8	1	0	9	7	5
9	7	8	5	4	6	3	0	1	2
4	1	7	0	9	5	8	2	6	3
8	0	3	1	5	4	2	7	9	6
1	9	2	3	6	7	5	4	0	8

28 seconds per ODLS pair
(**~1000 times faster**)

- Nested loops implementation?
- GPU/Phi implementation?

What we know about ESODLS of order 10?

- 32010 ESODLS CFs — Gerasim@Home results:
  - 30429 SODLS CFs — SOLS to SODLS (whitefox);
  - 1581 CFs — Gerasim@Home generalized symmetries neighborhoods.

Combinatorial structures:

ONCE (A):1 - 32010, where:
1 CFs - 32010

LINE4 (C):1 - 3, where:
2 CFs - 3

LINE4 (C):2 - 3, where:
2 CFs - 3

LOOP4 (E):2 - 76, where:
1 CFs - 2
2 CFs - 74

Computing experiment (from 07.2019)

Very rare objects!

0	1	2	3	4	5	6	7	8	9
1	2	0	6	7	9	8	3	4	5
3	6	7	9	8	4	2	5	1	0
4	0	8	5	2	3	7	1	9	6
5	9	4	8	3	6	0	2	7	1
7	8	6	4	0	1	3	9	5	2
6	4	5	2	1	7	9	0	3	8
9	5	1	7	6	0	4	8	2	3
2	3	9	0	5	8	1	4	6	7
8	7	3	1	9	2	5	6	0	4

06.08.2019

1CF Loop-4 (re-find in different WU’s)

0	1	2	3	4	5	6	7	8	9
1	2	0	4	5	7	9	8	6	3
5	0	1	6	3	9	8	2	4	7
9	3	5	8	2	1	7	4	0	6
4	6	3	5	7	8	0	9	2	1
8	4	6	9	1	3	2	5	7	0
7	8	9	0	6	4	5	1	3	2
2	9	4	7	8	0	3	6	1	5
6	5	7	1	0	2	4	3	9	8
3	7	8	2	9	6	1	0	5	4

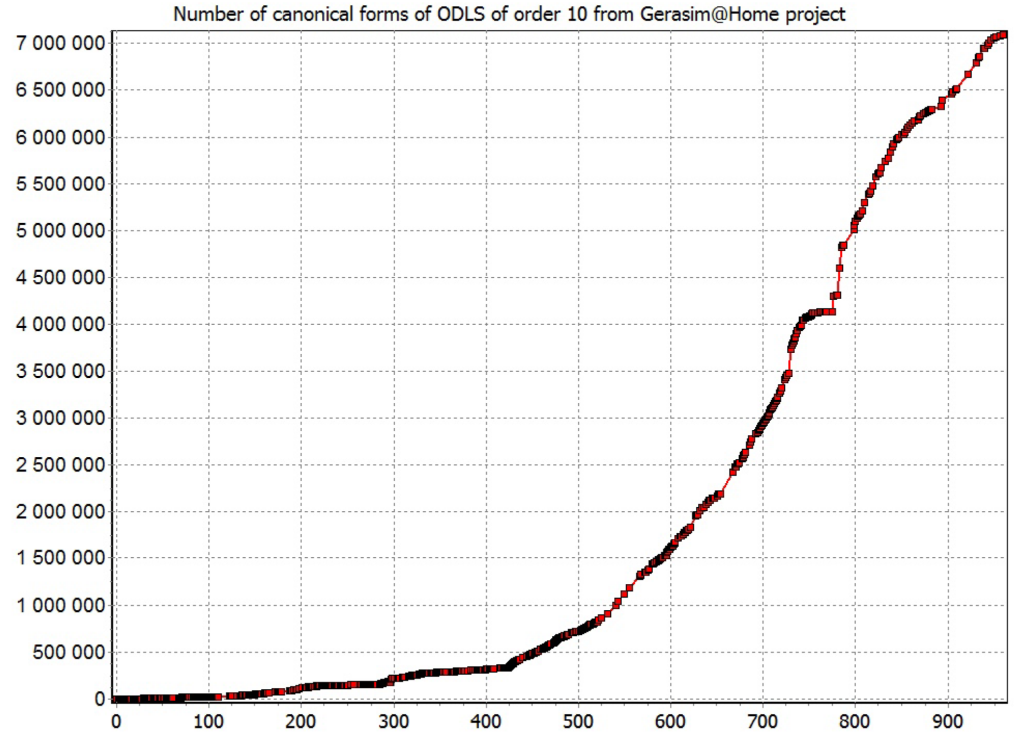
09.08.2019

1CF Once (**new!**)

- +4 additional 1 CF Onces (**new!**) and 13 re-find 1 CF Onces

Getting ODLS CFs within Gerasim@Home project

Strategy of search: getting source square (random generator, symmetric random generator), try to get orthogonal square, add the unique CF to collection



- Brute Force with bits arithmetic (03.2017)
- DLX v1, array (04.2017)
- DLX v2, pointers (05.2017)
- SN DLS (SCFs) (08.2017)
- horizontal symmetry (10.2017)
- different canonization strategy (04.2018)

Related work

Collecting CFs and new combinatorial structures search:

- triple of MODLS (is it exist?)
- different structures?

GPU implementation of transversal, cover and ESODLS algorithms?

Enumeration problems (OEIS):

- expanding current sequences
- enumerating DLS and ODLS of special kind (string-inverse, symmetric, ...) and its CFs

Pseudo triples:

- 3 kinds of pseudo triples, only 1 was investigated in details

Thank you for your attention!

Thanks to all the volunteers who took part in the Gerasim@home project!

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