QUALITY ANALYSIS OF BLOCK SEPARATIONS OF GRAPH SCHEMES OF PARALLEL CONTROL ALGORITHMS DURING LOGIC CONTROL SYSTEMS DESIGN USING GRID SYSTEMS ON VOLUNTEER BASIS

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GRID’16
Dubna, JINR, 2016
Parallel logic control algorithm

Development

Logic control system

Logic conditions
\[ X = x_1, x_2, \ldots, x_N \]

Control object

Microoperations
\[ Y = y_1, y_2, \ldots, y_M \]

Problems:
1. Separation
2. Allocation
3. Fault tolerance
4. Routing
5. Collective interactions etc.
Statement of a problem

\[ \bigcup_{i=1}^{H} A_i = A^0, \quad A_i \neq \emptyset, \quad A_i \cap A_j = \emptyset, \quad i, j = 1, H, \quad i \neq j, \]

\[ \neg (a_i \omega a_j) \quad \forall a_i, a_j \in A_k, \quad i \neq j, \quad k = 1, H, \]

\[ W(A_i) \leq W_{\text{max}}, \quad |X(A_i)| \leq X_{\text{max}}, \quad |Y(A_i)| \leq Y_{\text{max}}, \quad i = 1, H. \]

Discrete combinatorial optimization problem

NP-hard problem

Multicriteria optimization problem
**Quality criteria**

\[ H \rightarrow \min \]

\[ Z_1 = \sum_{i=1}^{H} \sum_{j=1, j \neq i}^{H} \alpha(A_i, A_j) \rightarrow \min \]

\[ Z_2 = \sum_{i=1}^{H} \sum_{j=1, j \neq i}^{H} \delta(A_i, A_j) \rightarrow \min \]

\[ Z_3 = \sum_{i=1}^{H} \left| X(A_i) - \left| X(A^0) \right| \right| \rightarrow \min \]

\[ Z_4 = \sum_{i=1}^{H} \left| Y(A_i) - \left| Y(A^0) \right| \right| \rightarrow \min \]
Example of separation (given with using PAE program system)
Methods of getting separations overview

Methods of getting separations:

- **Brute Force** (NP-hard problem, N<10 only);
- **S.I. Baranov method** (1984, greedy approach);
- **Greedy with sequential building of blocks and using adjacent vertices** (2013, greedy with additional constraints);
- A.D. Zakrevsky method (coloring of special graph);
- **Parallel-sequential method** (1997–Now, special method with transformations of graph-schemes of parallel algorithms);
- Well known discrete combinatorial optimization methods (stochastic methods: random search, directed random search, ACO, BC, GA, SA, etc.).

Quality of decisions, time and memory costs, complexity and features of practical implementation are significantly differ!

Which method is the best?
Quality analysis: parameters space and experiments

- fixed parameters experiments (2007), 10 minutes of computing time;
- variable value of 1 parameter (2008), 5 hours of computing time;
- variable values of 2 parameters (2010–Now), 430 years of computing time (with using BOINC).
Experimental results: expanding analyzed area
Example: prefer maps

- **red** – S.I. Baranov method,
- **green** – parallel-sequential method,
- **blue** – greedy adjacent strategy,
- **yellow** – random search.

Observed strong zone dependence of quality of decisions for different methods from area of parameters space!

What method is a best one? Answer depends from area of parameters space!
USING GREEDY METHODS ON VOLUNTEER BASIS FOR COMPARISON OF DECISIONS QUALITY OF HEURISTIC METHODS IN THE PROBLEM OF GETTING THE SHORTEST PATH IN THE GRAPH WITH GRAPH DENSITY CONSTRAINT

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Classification of methods for solving discrete combinatorial optimization problems

Universal methods:
- Brute Force (branches and bounds strategy);
- Limited Depth First Search;
- Greedy approach;
- (Weighted) Random Search;
- Ant Colony Optimization;
- Simulated Annealing;
- Genetic Algorithms.

Special methods:
- Parallel-Sequential Method;
- Dijkstra’s algorithm;
- Hungarian algorithm (Kuhn-Munkers algorithm);
- ...
Classification of methods for solving discrete combinatorial optimization problems

Precise methods (ensures an optimal solution):
- Brute Force (branches and bounds strategy);
- Dijkstra’s algorithm;
- Hungarian algorithm (Kuhn-Munkers algorithm);
- Greedy approach (Minimal spanning tree).

Approximate methods (ensures an sub- or quasi-optimal solutions):
- Limited Depth First Search;
- Greedy approaches (most tasks);
- (Weighted) Random Search;
- Ant Colony Optimization;
- Simulated Annealing;
- Genetic Algorithms;
- Different special methods.
Classification of methods for solving discrete combinatorial optimization problems

Consecutive methods:
- Greedy approach;
- Some special methods (Dijkstra’s algorithm, parallel-sequential method, ...).

Iterative methods:
- Brute Force (branches and bounds strategy);
- Limited Depth First Search;
- (Weighted) Random Search;
- Ant Colony Optimization;
- Simulated Annealing;
- Genetic Algorithms.
Assessment of the quality of decisions, convergence rate and computing time costs

Quality of decisions:
- during $C \rightarrow \infty$ $Q \rightarrow Q^*$;
- how to determine $C_{\text{min}}$ that $\Delta Q = |Q - Q^*|$ is acceptable for selected problem (for example, $\Delta Q/Q^*100\% < 5\%)$?

Ability of paralleling:
- Parallel execution is difficult or ineffective (for example, parallel-sequential method – theoretically no more than 10% speedup by Amdahl’s law);
- Weak or strong coupled implementations (AC with $M=1$ or $M=100$) – limiting classes of hardware;
- Trivial parallelized (random search, weighted random search).
Assessment of the quality of decisions based on samples of random source data

Sample of random source data

\[ \Lambda = \{G_1, G_2, ..., G_K\} \]

Average value of quality criteria

\[
\bar{Q} = \frac{\sum_{i=1}^{K} Q(G_i) \phi(G_i)}{K}, \quad \phi(G_i) \in \{0, 1\}
\]

Average deviation from optimum

\[
\Delta \bar{Q} = \frac{\sum_{i=1}^{K} (Q(G_i) - Q^*(G_i)) \phi(G_i)}{K}
\]
Assessment of the quality of decisions based on samples of random source data

Probability of finding decision

\[ \overline{p} = \frac{\sum_{i=1}^{K} \phi(G_i)}{K} \]

Probability of finding optimal decision

\[ \overline{p}_{opt} = \frac{\sum_{i=1}^{K} \theta(G_i)}{K}, \quad \theta(G_i) = \begin{cases} 0, & Q(G_i) > Q^*(G_i) \\ 1, & Q(G_i) = Q^*(G_i) \end{cases} \]

Average number of iterations (convergence rate)

\[ \overline{C} = \frac{\sum_{i=1}^{K} C(G_i)}{K} \]
Statement of a problem

\[G = \{A, V\}\]

\[A = \{a_1, a_2, \ldots, a_N\}\]

\[V = \{v_1, v_2, \ldots, v_M\}, \quad A \times A \subseteq V\]

\[d = \frac{M}{N(N-1)}\]

\[l(v_i) = l_{j,k}, \quad v_i = (a_i, a_k), \quad \exists l(v_i) = \infty\]

\[P = [a_{i_1}, a_{i_2}, \ldots, a_{i_m}]\]

\[L = \sum_{j=1}^{m-1} l_{i_j, i_{j+1}} \rightarrow \min\]
Simulated annealing: work on the bugs

- Short (1 week) experiment within Gerasim@Home
- Version 1 was adopted for wrong conditions during meta-optimization!
- No significant changes!
Total comparison of all methods

- AC/ACR
- AC
- RMR/
- ACR/

0 < N < Nmax

- o
- g
- rm
- wrm
- ac
- acr
- ldfs
- wrrm
- sa
- rrr
- gr
- lbf
- ac2
- acr2
- ga
- bc
• Convergence rate significantly differ for different methods!
Convergence rate analysis

- Methods can be combined?
Convergence rate analysis
Careful analysis: computing time costs

- Time depends from graph density!
- Long (ACR2) vs. Short (LBF) comparison: 3000 times differ!
- Time costs must be considered in the convergence rate experiments!
USING GRID SYSTEMS FOR ENUMERATING COMBINATORIAL OBJECTS ON EXAMPLE OF DIAGONAL LATIN SQUARES

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GRID’16
Dubna, JINR, 2016
Latin squares: what is it?

\[ A = \begin{bmatrix} a_{ij} \end{bmatrix} \]
\[ i, j = 1, N \]
\[ N = |S| \]
\[ S = \{0, 1, 2, ..., N - 1\} \]

\[ \forall i, j, k = 1, N, j \neq k: (a_{ij} \neq a_{ik}) \wedge (a_{ji} \neq a_{ki}) \]
\[ \forall i, j = 1, N, i \neq j: (a_{ii} \neq a_{jj}) \wedge (a_{N-i+1, N-j+1} \neq a_{N-j+1, N-i+1}) \]

<table>
<thead>
<tr>
<th>Latin square with order 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalized Latin square with order 10</td>
</tr>
<tr>
<td>( N! \times (N - 1)! )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Normalized Latin square with order 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \begin{bmatrix} 0 &amp; 1 &amp; 2 &amp; 3 &amp; 4 &amp; 5 &amp; 6 &amp; 7 &amp; 8 &amp; 9 \ 1 &amp; 2 &amp; 9 &amp; 4 &amp; 3 &amp; 6 &amp; 7 &amp; 8 &amp; 0 &amp; 8 \ 2 &amp; 9 &amp; 3 &amp; 1 &amp; 7 &amp; 0 &amp; 5 &amp; 8 &amp; 4 &amp; 6 \ 3 &amp; 4 &amp; 1 &amp; 2 &amp; 8 &amp; 7 &amp; 9 &amp; 6 &amp; 5 &amp; 0 \ 4 &amp; 3 &amp; 5 &amp; 9 &amp; 2 &amp; 1 &amp; 8 &amp; 0 &amp; 6 &amp; 7 \ 5 &amp; 6 &amp; 4 &amp; 8 &amp; 1 &amp; 2 &amp; 0 &amp; 9 &amp; 7 &amp; 3 \ 6 &amp; 5 &amp; 8 &amp; 7 &amp; 0 &amp; 3 &amp; 2 &amp; 1 &amp; 9 &amp; 4 \ 7 &amp; 8 &amp; 6 &amp; 0 &amp; 9 &amp; 4 &amp; 1 &amp; 2 &amp; 3 &amp; 5 \ 8 &amp; 7 &amp; 0 &amp; 5 &amp; 6 &amp; 9 &amp; 3 &amp; 4 &amp; 1 &amp; 2 \ 9 &amp; 0 &amp; 7 &amp; 6 &amp; 5 &amp; 8 &amp; 4 &amp; 3 &amp; 2 &amp; 1 \end{bmatrix} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>First string ordered diagonal Latin square with order 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \begin{bmatrix} 0 &amp; 1 &amp; 2 &amp; 3 &amp; 4 &amp; 5 &amp; 6 &amp; 7 &amp; 8 &amp; 9 \ 7 &amp; 2 &amp; 4 &amp; 9 &amp; 0 &amp; 6 &amp; 5 &amp; 1 &amp; 3 &amp; 8 \ 8 &amp; 3 &amp; 6 &amp; 7 &amp; 5 &amp; 9 &amp; 0 &amp; 2 &amp; 4 &amp; 1 \ 2 &amp; 6 &amp; 8 &amp; 5 &amp; 1 &amp; 7 &amp; 4 &amp; 0 &amp; 9 &amp; 3 \ 5 &amp; 8 &amp; 9 &amp; 1 &amp; 7 &amp; 0 &amp; 3 &amp; 4 &amp; 6 &amp; 2 \ 9 &amp; 4 &amp; 1 &amp; 2 &amp; 8 &amp; 3 &amp; 7 &amp; 6 &amp; 0 &amp; 5 \ 4 &amp; 7 &amp; 5 &amp; 6 &amp; 9 &amp; 1 &amp; 8 &amp; 3 &amp; 2 &amp; 0 \ 3 &amp; 0 &amp; 7 &amp; 8 &amp; 2 &amp; 4 &amp; 1 &amp; 9 &amp; 5 &amp; 6 \ 6 &amp; 5 &amp; 0 &amp; 4 &amp; 3 &amp; 2 &amp; 9 &amp; 8 &amp; 1 &amp; 7 \ 1 &amp; 9 &amp; 3 &amp; 0 &amp; 6 &amp; 8 &amp; 2 &amp; 5 &amp; 7 &amp; 4 \end{bmatrix} )</td>
</tr>
</tbody>
</table>
Getting Latin squares: enumerating, existence, but not optimizing combinatorial problem?

\[
A = \begin{pmatrix}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
9 & 0 & 6 & 1 & 5 & 3 & 8 & 2 & 7 & 4 \\
8 & 3 & 0 & 5 & 7 & 1 & 2 & 6 & 4 & - \\
2 & 4 & 5 & 9 & 1 & 0 & 7 & 8 & 6 & 3 \\
1 & 2 & 7 & 6 & 9 & 4 & 3 & 0 & 5 & 8 \\
7 & 8 & 4 & 2 & 0 & 9 & 5 & 1 & 3 & 6 \\
6 & 9 & 8 & 4 & 3 & 7 & 1 & 5 & 0 & 2 \\
4 & 6 & 9 & 8 & 2 & - & 0 & 3 & 1 & 5 \\
3 & 5 & 1 & 7 & 6 & 8 & 9 & 4 & 2 & 0 \\
5 & 7 & 3 & 0 & 8 & 2 & 4 & 9 & - & 1
\end{pmatrix}
\]

\[
S_{3,10}^L = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}
\]

\[
S_{3,10} = S \setminus (S_{3,10}^L \cup S_{3,10}^U) = \emptyset
\]

\[
C(A) = 3
\]

\[
S_{3,10}^U = \{4, 9\}
\]
Getting Latin squares: random search approach (RS)

\[ S_{ij} = \{s_{ij}^1, s_{ij}^2, \ldots, s_{ij}^M\} \subseteq S \]

\[ M(S_{ij}) = |S_{ij}| \]

\[ f_{RS}(s_{ij}^l) = r_k, l = 1, M(S_{ij}) \]

\[ C(A) \leq 1 \]

<table>
<thead>
<tr>
<th>(N)</th>
<th>The number of decisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>68</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

- No decisions found for \(N=10\) during 30 s CPU-time computing experiment (Intel Atom N270 @ 1.6 GHz, Diamondville core)
Abilities estimation

\[ f_{ij}^{(x)} = \sum_{k=j+1}^{N} |S_{ik}| + \sum_{k=i+1}^{N} |S_{kj}| = \sum_{k=j+1}^{N} |S_{ik}| + (N - i)|S_{i+1,j}| \to \max \]

\[ g_{ij}^{(x)} = f_{ij}^{(x)} + \alpha_{ij} \sum_{k=i+1}^{N} |S_{kk}| + \beta_{ij} \sum_{k=i+1}^{N} |S_{k,N-k}| \to \max \]

\[ \alpha_{ij} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases} \]

\[ \beta_{ij} = \begin{cases} 0, & i + j \neq N \\ 1, & i + j = N \end{cases} \]

\( a) \ x=1: f_{64}=5+2+3+3+1+4+4x4=34 \)

\( b) \ x=6: f_{64}=4+3+3+4+2+4+4x4=36 \)

\( S_{74} = \{2, 4, 6, 7\} \)

\( S_{84} = \{2, 4, 6, 7\} \)

\( S_{94} = \{2, 4, 6, 7\} \)

\( S_{10,4} = \{2, 4, 6, 7\} \)
Getting Latin squares: greedy approach (G)

\[ f_G(s^l_{ij}) = g(s^l_{ij}) \rightarrow \max, l = 1, M(S_{ij}) \]

<table>
<thead>
<tr>
<th>(N)</th>
<th>The number of violations</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

- No decisions found for \(N=3, 4, \ldots, 10\)
Getting Latin squares: weighted random search approach (WRS)

\[ f_{\text{WRS}}(s'_{ij}) = g_{ij}(s'_{ij}) \cdot (1 + 2d(r_k - 0.5)) \rightarrow \text{max}, \ l = 1, M(S_{ij}) \]

\[ C(A) \leq 1 \]

<table>
<thead>
<tr>
<th>( N )</th>
<th>RS</th>
<th>WRS</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>68</td>
<td>839</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- No decisions found for \( N=10 \) during 30 s CPU-time computing experiment (Intel Atom N270 @ 1,6 GHz, Diamondville core)
Getting Latin squares: ant colony optimization (AC)

\[ p_{ij} = \left[ g_{ij}^{(x)} \right]^\alpha \left[ T_{ij}^{(x)} \right]^\beta r_k \rightarrow \max \]

\[ f_{AC}(s_{ij}^l) = p_{ij}^{s_{ij}^l} \rightarrow \max, l = 1, M(S_{ij}) \]

\[ \Delta \tau = \frac{Q}{C(A_t) + 1} \]

\[ \alpha^* = 100, \]
\[ \beta^* = 1,1, \]
\[ \gamma^* = 0,99999, \]
\[ Q = 1,0 \]

Path elements

Graph vertices

- We get first decisions for N=10 during 30 s CPU-time computing experiment (Intel Atom N270 @ 1,6 GHz, Diamondville core)
Getting Latin squares: limited brute force (LBF)

\[ L_1 = (9 \ 7 \ 0 \ 6 \ 5 \ ? \ ? \ ? \ ? \ ?) - 132 \text{ ms per decision} \]

\[ L_1 = (2 \ 3 \ 1 \ 0 \ 5 \ ? \ ? \ ? \ ? \ ?) - 300 \text{ ms per decision} \]

AC preferences for first string:

\((9 \ 2 \ 0 \ 7 \ 8 \ 6 \ 4 \ 1 \ 3 \ 5),\)
\((9 \ 5 \ 0 \ 6 \ 8 \ 7 \ 3 \ 2 \ 1 \ 4),\)
\((9 \ 2 \ 0 \ 6 \ 8 \ 7 \ 3 \ 1 \ 4 \ 5),\)
\((9 \ 6 \ 0 \ 8 \ 7 \ 3 \ 1 \ 4 \ 2 \ 5),\)
\((9 \ 3 \ 0 \ 8 \ 7 \ 6 \ 2 \ 1 \ 4 \ 5).\)

<table>
<thead>
<tr>
<th>Method</th>
<th>(C(A) \leq 1)</th>
<th>(C(A) = 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>7 934</td>
<td>124</td>
</tr>
<tr>
<td>WRS</td>
<td>13 341</td>
<td>352</td>
</tr>
<tr>
<td>AC</td>
<td>179 387</td>
<td>9 422</td>
</tr>
<tr>
<td>LBF</td>
<td>–</td>
<td>346 572</td>
</tr>
</tbody>
</table>

- We get many decisions for \(N=10\) during **16 hours** CPU-time computing experiment (**Intel Core i7 4770 @ 3.4 GHz, Haswell core**)
- What pace of generation? 3–7 DLS/s!
### Let's try to improve pace! Diagonals first fill

<table>
<thead>
<tr>
<th>Method</th>
<th>Sequential fill</th>
<th>Diagonals first fill</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>≈ 0 DЛК/с (0,4 DLS/s with C=1)</td>
<td>0,5 DЛК/с (18 DLS/s with C=1)</td>
<td>45x</td>
</tr>
<tr>
<td>WRS</td>
<td>≈ 0 DЛК/с (0,7 DLS/s with C=1)</td>
<td>0,8 DЛК/с (16 DLS/s with C=1)</td>
<td>23x</td>
</tr>
<tr>
<td>AC</td>
<td>≈ 0 DЛК/с (0,05 DLS/s with C=1)</td>
<td>0,14 DLS/s</td>
<td>–</td>
</tr>
<tr>
<td>LBF</td>
<td>≈ 0 DLS/s</td>
<td>28 DLS/s</td>
<td>–</td>
</tr>
</tbody>
</table>

- **Pace is 28 DLS/s!**
- **Only for DLS, not for LS!**
Diagonals first fill with out of order fill

$|S_{ij}| = 1$

- Universal principle, can be used in different combinatorial problems!
### Diagonals first fill with out of order fill

$$|S_{ij}| = 1$$

<table>
<thead>
<tr>
<th>Method</th>
<th>Without out of order fill</th>
<th>With out of order fill</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>1,0 DLS/s</td>
<td>164 DLS/s</td>
<td>164x</td>
</tr>
<tr>
<td>WRS</td>
<td>3,1 DLS/s</td>
<td>203 DLS/s</td>
<td>65x</td>
</tr>
<tr>
<td>AC</td>
<td>0,2 DLS/s</td>
<td>224 DLS/s</td>
<td>1120x</td>
</tr>
<tr>
<td>BF</td>
<td>363 DLS/s</td>
<td>13 000 – 15 000 DLS/s</td>
<td>36x – 41x</td>
</tr>
</tbody>
</table>

- **Pace is 15 000 DLS/s!**
Fast checking of ability sets

\[ S_{ij} = U \setminus \bigcup_{k=1}^{N} \{ a_{ik} \} \setminus \bigcup_{k=1}^{N} \{ a_{kj} \} \bigcup_{k=1}^{N} \{ a_{kk} \} \setminus \bigcup_{k=1}^{N} \{ a_{k,N-k} \}, \]

only for main diagonal elements with \( i=j \)
only for second diagonal elements with \( i+j=N \)

\[ s_i = \bigcup_{k=1}^{N} \{ a_{ik} \} \quad r_j = \bigcup_{k=1}^{N} \{ a_{kj} \} \quad d_1 = \bigcup_{k=1}^{N} \{ a_{kk} \} \quad d_2 = \bigcup_{k=1}^{N} \{ a_{k,N-k} \} \]

<table>
<thead>
<tr>
<th>Method</th>
<th>Without fast checking</th>
<th>With fast checking</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>164 DLS/s</td>
<td>662 DLS/s</td>
<td>4x</td>
</tr>
<tr>
<td>WRS</td>
<td>203 DLS/s</td>
<td>740 DLS/s</td>
<td>3,6x</td>
</tr>
<tr>
<td>AC</td>
<td>224 DLS/s</td>
<td>781 DLS/s</td>
<td>3,5x</td>
</tr>
<tr>
<td>BF</td>
<td>13 000 – 15 000 DLS/s</td>
<td>38 000 DLS/s</td>
<td>2,5x – 2,9x</td>
</tr>
</tbody>
</table>

- Pace is **38 000 DLS/s**!
Excluding background CPU load (Hyper-Threading)

Gerasim@Home and SAT@Home through BOINC – background CPU load must be excluded!

<table>
<thead>
<tr>
<th>Method</th>
<th>Before</th>
<th>After</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>662 DLS/s</td>
<td>1 040 DLS/s</td>
<td>1,6x</td>
</tr>
<tr>
<td>WRS</td>
<td>740 DLS/s</td>
<td>1 130 DLS/s</td>
<td>1,5x</td>
</tr>
<tr>
<td>AC</td>
<td>781 DLS/s</td>
<td>1 190 DLS/s</td>
<td>1,5x</td>
</tr>
<tr>
<td>BF</td>
<td>38 000 DLS/s</td>
<td>56 000 DLS/s</td>
<td>1,5x</td>
</tr>
</tbody>
</table>

- Pace is **56 000 DLS/ s** for single-threaded program!
Clipping and early combinatorial returns

\[ \begin{array}{cccc}
0 & 1 & 2 & 3 & 4 \\
3 & 2 & 1 &  & \\
 & 3 & & & \\
0 & 4 & & & \\
2 & & 1 & & \\
\end{array} \]

\[ \begin{array}{cccc}
0 & 1 & 2 & 3 & 4 \\
3 & 2 & 1 & 0 & \\
 & 4 & 3 & 2 & \\
1 & 0 & 4 & & \\
2 & & 1 & & \\
\end{array} \]

\[ \begin{array}{cccc}
0 & 1 & 2 & 3 & 4 \\
3 & 2 & 1 & 0 & \\
 & 4 & 3 & 2 & \\
1 & 0 & 4 & & \\
2 & & 1 & & \\
\end{array} \]

<table>
<thead>
<tr>
<th>Method</th>
<th>Without</th>
<th>With</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>1 040 DLS/s</td>
<td>1 040 DLS/s</td>
<td>–</td>
</tr>
<tr>
<td>WRS</td>
<td>1 130 DLS/s</td>
<td>1 170 DLS/s</td>
<td>+3.5%</td>
</tr>
<tr>
<td>AC</td>
<td>1 190 DLS/s</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>BF</td>
<td>56 000 DLS/s</td>
<td>101 000 DLS/s</td>
<td>1.8x</td>
</tr>
</tbody>
</table>

- AC with returns \(\rightarrow\) WRS with returns
- Pace is **101 000 DLS/s** for single-threaded program!
Pace is **200 000 DLS/ s** for recurrent Delphi program

Pace is **217 000 DLS/ s** for recurrent C++ program (VS2012)

Pace is **240 000 DLS/ s** for recurrent C++ program (VS2012 + PGO)
Diagonals first fill with none sequential fill with filled elements

for (LS[77] = 0; LS[77]< N; LS[77]++) {
    if ((Strs[7][LS[77]] || !Rows[7][LS[77]] || !d1[LS[77]])
        continue;

    Strs[7][LS[77]] = 0;
    Rows[7][LS[77]] = 0;
    d1[LS[77]] = 0;
}

for (LS[33] = 0; LS[33]< N; LS[33]++) {
    if ((Strs[3][LS[33]] || !Rows[3][LS[33]] || !d1[LS[33]])
        continue;

    Strs[3][LS[33]] = 0;
    Rows[3][LS[33]] = 0;
    d1[LS[33]] = 0;
}

    if ((Strs[3][LS[36]] || !Rows[3][LS[36]] || !d2[LS[36]])
        continue;

    Strs[3][LS[36]] = 0;
    Rows[3][LS[36]] = 0;
    d2[LS[36]] = 0;
}

for (LS[63] = 0; LS[63]< N; LS[63]++) {
    if ((Strs[6][LS[63]] || !Rows[6][LS[63]] || !d2[LS[63]])
        continue;

    Strs[6][LS[63]] = 0;
    Rows[3][LS[63]] = 0;
    d2[LS[63]] = 0;
}

for (LS[66] = 0; LS[66]< N; LS[66]++) {
    if ((Strs[6][LS[66]] || !Rows[6][LS[66]] || !d1[LS[66]])
        continue;

    Strs[6][LS[66]] = 0;
    Rows[6][LS[66]] = 0;
    d1[LS[66]] = 0;
}

• Pace is **212 000 DLS/s** for iterative Delphi program
• Pace is **340 000 DLS/s** for iterative C++ program (VS2012 + PGO)
SAT-generation of DLSs

<table>
<thead>
<tr>
<th>Number of DLS</th>
<th>Time</th>
<th>Pace</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 000</td>
<td>0.34 s</td>
<td>2 941 DLS/s</td>
</tr>
<tr>
<td>10 000</td>
<td>8.528 s</td>
<td>1 173 DLS/s</td>
</tr>
<tr>
<td>100 000</td>
<td>504.468 s</td>
<td>198 DLS/s</td>
</tr>
</tbody>
</table>

- Pace decreased during increasing number of restrictions
Conclusion and prospects of future work

1. Current pace is **1 800 000 DLS/s**. This is end? GPU?
2. We can enumerate DLS and MOLS (LS is enumerated: OEIS A000479, A000315).
3. We can find all orthogonal pairs of DLS and MOLS for small $N$.
4. We can organize parameters space exploration with heuristic methods, local analysis with LBF and find some orthogonal pairs of DLS for big $N$.
5. We can get isomorphism classes and canonical forms for DLS.
6. This problems are weakly coupled and can be solved with volunteer computing support!

- Vatutin E.I., Zhuravlev A.D., Zaikin O.S., Titov V.S. Using algorithmic features in the problem of generating diagonal Latin squares (in Russian) // Proceedinds of Southwest State University. Accepted for publication
- Vatutin E.I., Zaikin O.S., Zhuravlev A.D., Manzuk M.O., Kochemazov S.E., Titov V.S. On the effect of the order of cells filling to the rate of generation of diagonal Latin squares // Diagnostics 2016. Accepted for publication
The End.
Thanks!

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