Basic definitions

DLS – diagonal Latin square (in this document of order 10).

Orthogonal DLSs (ODLS) – pair of DLS A and B, in which all ordered pairs of elements $\left(a_{ij}, b_{ij}\right)$ are distinct.

String representation of DLS – elements of DLS that are written from left to right from top to bottom:

3 2 8 4 6 7 1 0 9 5
8 1 2 7 4 6 3 5 0 9
1 5 0 9 8 2 4 3 7 6
6 8 5 2 0 9 7 1 4 3
9 0 7 1 5 4 2 6 3 8
4 3 9 0 1 8 6 7 5 2
0 6 3 8 7 5 9 2 1 4
5 7 6 3 9 1 8 4 2 0
7 9 4 5 2 3 0 8 6 1
2 4 1 6 3 0 5 9 8 7

3284671095812746350915098243766852097143907154263843901867520638759214576391842079452308612416305987

DLX – Dancing Links X algorithm – algorithm for exact cover problem solving.

Canonical form (CF) – lexicographically minimal string representation of DLS within corresponding isomorphism (isotopism) class.

Symmetry – some correspondence between elements of DLS. Can be viewed geometrically (vertical or horizontal plane symmetry, central symmetry) or generalized through structure of lengths of loops of permutations $\left(P_X, P_Y, P_V\right)$. For $N = 10$ well known $10! = 3\,628\,800$ different permutations but only 42 from them has different structures of loops.

Partially symmetric DLS – DLS, in which only M cells corresponds to some symmetry and another cells has asymmetric filling, in correspondence with definition of correct DLS.

Combinatorial structure – graph, in which vertices are DLSs and edges are corresponds to the orthogonality binary relationship between pair of DLSs. The set of different combinatorial structures that are found within distributed computing project Gerasim@Home is shown below.

1. DLS without ODLS (bachelor)

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 0123456789120453986735618740929478321605273098514668592473104697013258704619852383156029745982760431 (DLS A, CF 1).

Adjacency matrix:

$M = (0)$.

Different CFs set within combinatorial structure:

<u>CF 1:</u> 0123456789120453986735618740929478321605273098514668592473104697013258704619852383156029745982760431 (DLS A).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 1, a = 0, \rho = [1]$.

Method of finding:

any type of search due to most amount of DLSs hasn’t ODLS.

2.1. ODLS pair without symmetry (asymmetric once)

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 0123456789120436597820156348973457981620574980216369812473058376019254963072854175681904324892573016 (DLS *A*, CF 1);

DLS 2: 0123456789375869241049871032561209745863941258063720943685716830271945854601739256719340287365829104 (DLS *B*, CF 2).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120436597820156348973457981620574980216369812473058376019254963072854175681904324892573016 (DLS *A*);

CF 2: 0123456789123067984589740613523759248016456812390776025841939847315260241593067850867924316391807524 (DLS *B*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 2, a = 1, \rho = [1, 1]$.

Method of finding:

any type of search in combination with method of checking DLS for ODLS (Euler-Parker method, DLX [9]), approximately among 30 millions of DLS exists only 1 DLS with ODLS.

2.2. ODLS pair with symmetry (symmetric once)

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 0123456789120458793654378916206798130254937562804189407135623651249807708296541325693041784816072395 (DLS *A*, CF 1);

<u>DLS 2:</u> 0123456789935786042125610748934609217538378654291072943851068945701362581063927410789236456432198057 (DLS *B*, CF 1).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

<u>CF 1:</u> 0123456789120458793654378916206798130254937562804189407135623651249807708296541325693041784816072395 (DLS *A*, DLS *B*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 2, a = 1, \rho = [1, 1]$.

Method of finding:

Brute Force for self-orthogonal ODLSs (SODLS) [1, 2] (symmetry provided by DLS transposing from the main diagonal or main anti-diagonal); also found different ODLS with same property for different *M*-transformations [3].

3.1. Line-3 without symmetry (asymmetric twice)

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 01234567891204369857278693510480571923463869571420597104863276482 1309545306279189415780263639280457 1 (DLS B, CF 1);
<u>DLS 2:</u> 01234567899652017438527834901638195246071905268374603478259143918 7526074869301258567103942274069185 3 (DLS A, CF 2);
<u>DLS 3:</u> 01234567899652817430527034981638195246071985260374603478259143910 7526874069381258567103942274869105 3 (DLS C, CF 3).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

<u>CF 1:</u> 01234567891204369857278693510480571923463869571420597104863276482130 95453062791894157802636392804571 (DLS B);
<u>CF 2:</u> 01234567891204789356437680129550892176432517648930769813540239605748 21873192056498453620176452093178 (DLS A);
<u>CF 3:</u> 01234567891204789356437980126550862179432517948630769813540239605748 21873162059498453620176452093178 (DLS C).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 3, a = 2, \rho = [1, 1, 2]$.

Method of finding:

any type of search in combination with method of checking DLS for ODLS (Euler-Parker method, DLX), approximately for 500–1000 onces it is present 1 asymmetric twice.

Remark:

line-3 structure also can be treated as «1:2 structure» or as rhombus-1.

3.2. Line-3 with symmetry (symmetric twice)

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 01234567891204365978203581469746971820359786543120347809125669417 28503785063941253192708648562907341 (DLS *B*, CF 1) – horizontal symmetry;

DLS 2: 01234567893857290416479836152092865031746510724893564281903783049 75261246918730510756329487931048652 (DLS *A*, CF 2);

DLS 3: 01234567893859072416974836102552869431706015729843269081753483742 05961496218035715076342987431598602 (DLS *C*, CF 2).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120436597820358146974697182035978654312034780912566941728 503785063941253192708648562907341 (DLS *B*);

CF 2: 0123456789120436895757169038423897524016645981027393782415608560179 324493278560126450971387081632495 (DLS *A*, DLS *C*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 3, a = 2, \rho = [1, 1, 2]$.

Method of finding:

any type of symmetric DLS search (plane symmetry [4–6], generalized symmetry [7]) in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

4.1. Line-4 without symmetry

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 0123456789120457983654682031972675138940678932405138107952647946081325935786041280916425734532917608 (DLS *B* , CF 1);

<u>DLS 2:</u> 0123456789731594862036597840124298601573987216340550863729411407239856694051723825348901678761025394 (DLS *A* , CF 2);

<u>DLS 3:</u> 0123456789731598462036597480124298601573947216380550863729411807239456694051723825348901678761025394 (DLS *C* , CF 3);

<u>DLS 4:</u> 0123456789120453987674682015932615378940658972403138701952645946083127973186045280976423154352917608 (DLS *D*, CF 4).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

<u>CF 1:</u> 0123456789120457983654682031972675138940678932405138107952647946081325935786041280916425734532917608 (DLS *B*);

<u>CF 2:</u> 0123456789123069584760578431925419082376284170963579683105244796238051937256140835841279608605974213 (DLS *A*);

<u>CF 3:</u> 0123456789120456893735719248069486307152593281067487690423156895173420704829156346107352982357689041 (DLS *C*);

<u>CF 4:</u> 0123456789120453896739872015468015947623749261503858760932146538724190964187230543501698722769380451 (DLS *D*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 4, a = 3, \rho = [1, 1, 2, 2]$.

Method of finding:

search for partially generalized symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

4.2. Line-4 with symmetry

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 0123456789123867094579640835124619702853380251946785973412062450867391908623517457419286306375194028 (DLS *A*, CF 1);

<u>DLS 2:</u> 0123456789769214835014367908259085327614576423109839706851428341579206250891346762198045734857062931 (DLS *B*, CF 2) – SODLS;

<u>DLS 3:</u> 0123456789123869047597640835124619702853380251496785973412062950867341708623519454719286306345179028 (DLS *C*, CF 2) – SODLS;

<u>DLS 4:</u> 0123456789765914832014367208952085397614976423105839706851428341579206520891346765128049734897062531 (DLS *D*, CF 1).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

<u>CF 1:</u> 0123456789120453897626897051437836124095675098132484950176324972863501356827941093176402585041392867 (DLS *B*, DLS *C*);

<u>CF 2:</u> 0123456789123867094579640835124619702853380251946785973412062450867391908623517457419286306375194028 (DLS *A*, DLS *D*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 4,\ a = 3,\ \rho = [1, 1, 2, 2]$.

Method of finding:

Brute Force of the SODLS.

5. Line-5 with symmetry

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 0123456789120436597823491805678765904321948263715058302796147698541032351709284640768132956951728403 (DLS *C*, CF 3) – horizontal symmetry;

DLS 2: 0123456789357809214672659043184081237695693478520187591204639342618570140687395226105498375897361024 (DLS *D*, CF 2);

DLS 3: 0123456789358709124618659043724032678195897412560363597804219241837560740621395826105498375798362014 (DLS *B*, CF 2);

DLS 4: 0123456789120436597823491805678765904321947263815058302796147698541032351709284640867132956951827403 (DLS *E*, CF 1);

DLS 5: 0123456789120436597823491805678765904321948163725058302796147698541032351709284640768231956952718403 (DLS *A*, CF 1).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120436597823491805678765904321947263815058302796147698541032351709284640867132956951827403 (DLS *A*, DLS *E*);

CF 2: 0123456789120479563834968120576075184392285736941085602739414638901275731954082697410285635982637104 (DLS *B*, DLS *D*);

CF 3: 0123456789120436597823491805678765904321948263715058302796147698541032351709284640768132956951728403 (DLS *C*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 5, a = 4, \rho = [1, 1, 2, 2, 2]$.

Method of finding:

Brute Force for horizontally symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

6.1. Loop-4 (1 different CF)

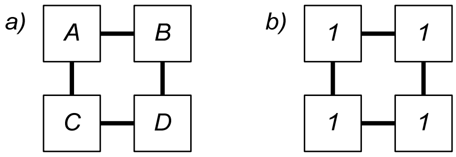

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 0123456789120679834536798425104085237196594836027178640139526452179038951760482323905814678731925604 (DLS A, CF 1);

DLS 2: 0123456789538907142615926378049674185230745021396832487690152706948153693182054780153946724867502391 (DLS B, CF 1);

DLS 3: 0123456789835907142615926378049674185230748021396532457690182706948153693182054750183946724867502391 (DLS C, CF 1);

DLS 4: 0123456789120679834563798425104085267193594863027178640139523452179068951730482626905814378731925604 (DLS D, CF 1).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120679834536798425104085237196594836027178640139526452179038951760482323905814678731925604 (DLS A, DLS B, DLS C, DLS D).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 4$, $a = 4$, $\rho = [2, 2, 2, 2]$.

Method of finding:

search for partially symmetric DLSs («+–»-symmetry, symmetric filled cells number $M = 70$ [7]) in combination with method of checking DLS for ODLS ODLS (Euler-Parker method, DLX).

Remark:

loop-4 structure also can be treated as rhombus-2.

6.2. Loop-4 (2 different CFs)

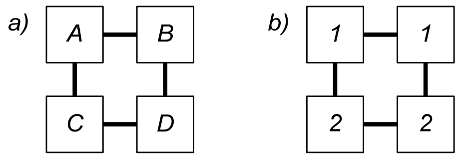

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 0123456789329601784578306951249347208651268593041719785432064562781093840136957257198243606054172938 (DLS *A*, CF 1) – SODLS;
<u>DLS 2:</u> 0123456789547193862043167290581902874536609438217585301679429287015364275964081376485032913865291407 (DLS *B*, CF 1) – SODLS;
<u>DLS 3:</u> 0123456789541793862043761290587902814536609438217585307619429281075364275964081316485032973865297401 (DLS *C*, CF 2) – SODLS;
<u>DLS 4:</u> 0123456789829601734578306951249347208651268593041719785432064562781093340186957257193248606054172938 (DLS *D*, CF 2) – SODLS.

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

<u>CF 1:</u> 0123456789120657893460947325189745261803453980712659806134727358124690867294035134170892652861395047 (DLS *A*, DLS *B*);
<u>CF 2:</u> 0123456789120467895395162430783687194205674893051240957826317931025864846950132758723691402350817496 (DLS *C*, DLS *D*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 4, a = 4, \rho = [2, 2, 2, 2]$.

Method of finding:

Brute Force of SODLS.

6.3. Loop-4 (3 different CFs)

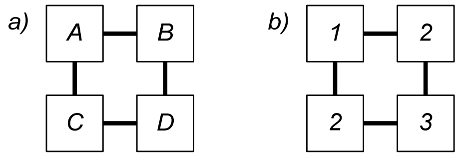

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 0123456789120463597820351846973847902516798654310264798102538560729341461209783593582714605791368024 (DLS *A*, CF 1) – horizontal symmetry;

DLS 2: 0123456789893726450175896401321064725398345018927642983710659876532410534190862767028139542615097843 (DLS *B*, CF 2);

DLS 3: 0123456789894537260176895301421064725398327018945643982610759857643210273190856454068179236512094837 (DLS *C*, CF 2);

DLS 4: 0123456789120463597820358146973847902516791654380264791802538560729341468209713593512784605798361024 (DLS *D*, CF 3) – horizontal symmetry.

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120463597820351846973847902516798654310264798102538560729341461209783593582714605791368024 (DLS *A*);

CF 2: 0123456789123058796434791608525861972043960483521769520483718097613425231679450875483216904785209136 (DLS *B*, DLS *C*);

CF 3: 0123456789120463597820315486975698271034486072931583520974617915364802658790214394761832503749810526 (DLS *D*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 4, a = 4, \rho = [2, 2, 2, 2]$.

Method of finding:

Brute Force for horizontally symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

6.4. Loop-4 (4 different CFs)

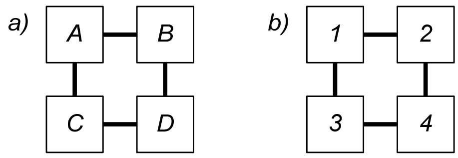

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 0123456789120437956878952430168056792341468753190269328154709761084253357916082424189076355340628197 (DLS *A*, CF 1);

<u>DLS 2:</u> 0123456789247659830182501796343947615028936182745050897621436514203897473208196578953402161608934572 (DLS *B*, CF 2);

<u>DLS 3:</u> 0123456789247659830182501796343947615028936182745050897621437514203896463208197568953402171708934562 (DLS *C*, CF 3);

<u>DLS 4:</u> 0123456789120437956878952430168056792341461753890269321854709768014253357986012424819076355340621897 (DLS *D*, CF 4).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

<u>CF 1:</u> 0123456789120437956878952430168056792341468753190269328154709761084253357916082424189076355340628197 (DLS *A*);

<u>CF 2:</u> 0123456789120453896726709851345489270613395682704165987134209015364872784160239543670912588732149506 (DLS *B*);

<u>CF 3:</u> 0123456789120497863567308251945617392048437150982639426875109068714352285604197384951302677589263401 (DLS *C*);

<u>CF 4:</u> 0123456789120437956878952430168056792341461753890269321854709768014253357986012424819076355340621897 (DLS *D*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 4, a = 4, \rho = [2, 2, 2, 2]$.

Method of finding:

search for partially generalized symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

7. Treshka (1:3 structure)

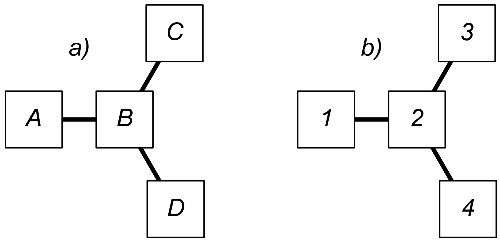

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 0123456789123709854640963718259684513072596870243134592806178701635294237596410875128493606840127953 (DLS *B*, CF 1);
<u>DLS 2:</u> 0123456789695178043292401685733576902841863951420748176239501364897025578204139620983756147405239168 (DLS *A*, CF 2);
<u>DLS 3:</u> 0123456789930487216512589370468497065321207564891345367192087962184530671920385458403216973681590472 (DLS *C*, CF 3);
<u>DLS 4:</u> 0123456789937480216512589370468497065321270564891345367192087962184530601927385458403216973681590472 (DLS *D*, CF 4).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

<u>CF 1:</u> 0123456789123709854640963718259684513072596870243134592806178701635294237596410875128493606840127953 (DLS *A*);
<u>CF 2:</u> 0123456789120453896760852973144516982073349271560876398401522960173845984762153087513042965378069421 (DLS *B*);
<u>CF 3:</u> 0123456789120468793593625081743791064258854972130654378906126980135427405627389126783195407815942063 (DLS *C*);
<u>CF 4:</u> 0123456789120468793595623081743791064258834972150654378906126980135427405627389126785193407815942063 (DLS *D*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 4, a = 3, \rho = [1, 1, 1, 3]$.

Method of finding:

horizontally symmetric DLSs canonization, search for partially generalized symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

8.1. Symmetric four (1:4 structure)

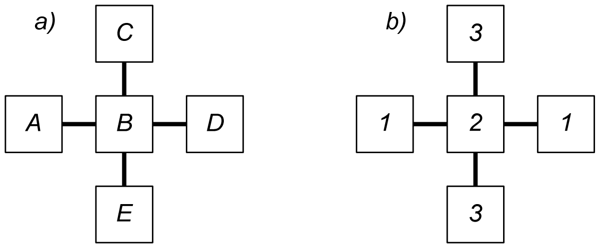

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 0123456789120436597823510984678596723041374918052640682713957480639152691254780396758142305837902614 (DLS *B*, CF 1) – horizontal symmetry;

DLS 2: 0123456789485027961360371825949612547830826590347117043659283976814205538972014674916380522548091367 (DLS *A*, CF 2);

DLS 3: 0123456789485927061369371825049612547830826590347117943650283076814295538072914674016389522548091367 (DLS *C*, CF 3);

DLS 4: 0123456789683027941550471826939612547830825690437117043659284975813206358972016474916380522368091547 (DLS *D*, CF 2);

DLS 5: 0123456789683927041559471826039612547830825690437117943650284075813296358072916474016389522368091547 (DLS *E*, CF 3).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120436597823510984678596723041374918052640682713957480639152691254780396758142305837902614 (DLS *B*);

CF 2: 0123456789120467983530981645272836791054795132840693158472604780935612867950214354620839716547210398 (DLS *A*, DLS *D*);

CF 3: 0123456789120457896390316278544765982031389671524079183604252380194576657984310284520316975647209318 (DLS *C*, DLS *E*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 5, a = 4, \rho = [1, 1, 1, 1, 4]$.

Method of finding:

search for horizontally symmetric and partially generalized symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

Remark:

first time horizontally symmetric string-inverse DLS corresponding for 1:4 structure was described in [4], but this article has only 3 ODLS for them; 4-th ODLS was found and described in article [8].

8.2. Asymmetric four (1:4 structure)

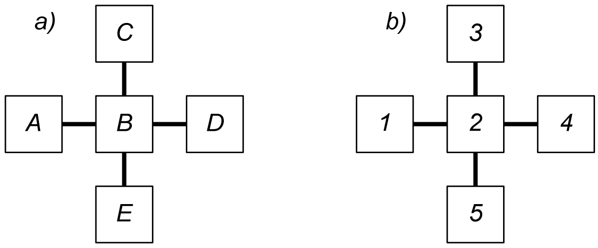

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 01234567891204379568348509217657392648019561708243784651039260529
83417297084163543186279508697135024 (DLS *A*, CF 1);

<u>DLS 2:</u> 01234567899546287031603754981243587126903602974158278516390478906
21345826109547314798305265914308267 (DLS *B*, CF 2);

<u>DLS 3:</u> 01234567899546187032603754982143587216903602974158178526390478906
12345826109547324798305165914308267 (DLS *C*, CF 3);

<u>DLS 4:</u> 01234567894596287031603754981293587126403602974158278516390478406
21395826109547314798305265914308267 (DLS *D*, CF 4);

<u>DLS 5:</u> 01234567894596187032603754982193587216403602974158178526390478406
12395826109547324798305165914308267 (DLS *E*, CF 5).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

<u>CF 1:</u> 0123456789120437956834850921765739264801956170824378465103926052983
417297084163543186279508697135024 (DLS *A*);

<u>CF 2:</u> 0123456789120453796846872935019815672430609231584785710642935438709
612374698012523591480767960821354 (DLS *B*);

<u>CF 3:</u> 0123456789120453896727586903419687214530643517980239460872154892703
156501984267375603214988371965024 (DLS *C*);

<u>CF 4:</u> 0123456789120458796346872935019815672430609231584735710642985438709
612874693012523591480767960821354 (DLS *D*);

<u>CF 5:</u> 0123456789120453896727586903419867214530643517980239460872154692703
158501984267375803214968371965024 (DLS *E*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 5, a = 4, \rho = [1, 1, 1, 1, 4]$.

Method of finding:

search for partially generalized symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

9. Five (1:5 structure)

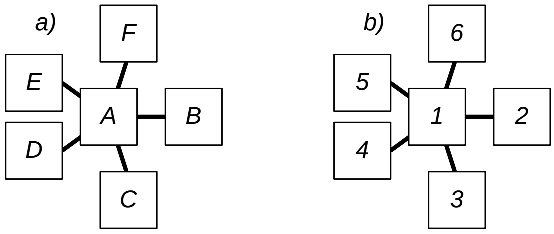

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 0123456789120487956393176804253795048216258693710479482613508631504972647032589140597126385862193047 (DLS *A*, CF 1);
DLS 2: 0123456789859603427167325918405489712063786410593292573806141945627308201896345736708491254301278596 (DLS *B*, CF 2);
DLS 3: 0123456789359608427167825913405439712068786410593292578306141945627803201896345786703491254301278596 (DLS *C*, CF 3);
DLS 4: 0123456789359608427167825931405419732068786430591292578106341945627803203896145786701493254301278596 (DLS *D*, CF 4);
DLS 5: 0123456789379608425165827913407459312068586410397292758306141947625803201896743586305491274301278596 (DLS *E*, CF 5);
DLS 6: 0123456789379608425165827931407459132068586430197292758106341947625803203896741586105493274301278596 (DLS *F*, CF 6).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120487956393176804253795048216258693710479482613508631504972647032589140597126385862193047 (DLS *A*);
CF 2: 0123456789123067594896821043578574032196749538162048165930723961847205230796851457492108636058729431 (DLS *B*);
CF 3: 0123456789123067594896871043528574032196249538167048165930273961847205730296851457492108636058729431 (DLS *C*);
CF 4: 0123456789120453796839708651242489670513906732485156487132906395108472853124960778520913464716982035 (DLS *D*);
CF 5: 0123456789120657893440372658916481927350935871042638790426155614809273796018354285423910672795634108 (DLS *E*);
CF 6: 0123456789120458796330627458919647801325538917264065183902744870923516795106843284362190572795634108 (DLS *F*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 6, a = 5, \rho = [1, 1, 1, 1, 1, 5]$.

Method of finding:

search for partially central symmetric DLSs, symmetric filled cells number $M = 60$, in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

10.1. Asymmetric six (1:6 structure)

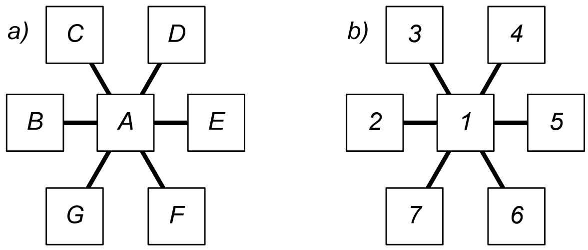

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 0123456789658209741328916350477308562194891734025640598713623276914508143528967097641038255640728931 (DLS *A*, CF 1);
DLS 2: 0123456789120678394553489072614639821507756213809429175604388490675312907431285638512496706785094123 (DLS *B*, CF 2);
DLS 3: 0123456789180672394553429078614639281507756813209429175604388490675312907431825632518496706785094123 (DLS *C*, CF 3);
DLS 4: 0123456789180627394553479028614639781502256813709479125604388490625317907431825632518496706785094123 (DLS *D*, CF 4);
DLS 5: 0123456789120678934559483072614691823507756291803423175604988430675912907413285638592416706785094123 (DLS *E*, CF 5);
DLS 6: 0123456789180672934559423078614691283507756891203423175604988430675912907413825632598416706785094123 (DLS *F*, CF 6);
DLS 7: 0123456789180627934559473028614691783502256891703473125604988430625917907413825632598416706785094123 (DLS *G*, CF 7).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789123406897526953840176487531290795084213640762198533718695402956172034883429075615809173624 (DLS *A*);
CF 2: 0123456789120678394553489072614639821507756213809429175604388490675312907431285638512496706785094123 (DLS *B*);
CF 3: 0123456789123450986779518624034368721095307968415224903156786842937510850627394197851402365617098324 (DLS *C*);
CF 4: 0123456789123450986779418625035368721094307968415224903156786852937410850627394197851402364617098325 (DLS *D*);
CF 5: 0123456789120678934559483072614691823507756291803423175604988430675912907413285638592416706785094123 (DLS *E*);
CF 6: 0123456789123470986570523614984796528031397081564286192745039547683120240813795663859402175861092374 (DLS *F*);

CF 7: 0123456789123078495647958026135418397062238461950796710482357062135 4988547961320680952317439562708410 (DLS *G*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 7, a = 6, \rho = [1, 1, 1, 1, 1, 1, 6]$.

Method of finding:

search for partially central symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

10.2. Symmetric six (1:6 structure)

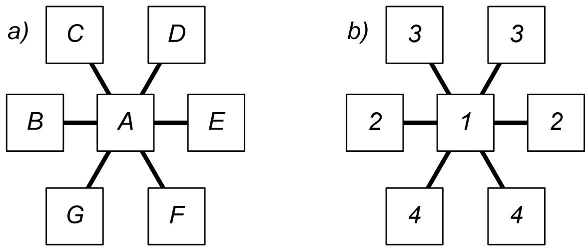

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 0123456789123409567823408195679876543210876590432164912780533508721946501763289476591804324982367105 (DLS *A*, CF 1) – horizontal symmetry, string-inverse square;

<u>DLS 2:</u> 0123456789846517293060972381542751864093164932087532145896075386097412490271356898706453217538901246 (DLS *B*, CF 2);

<u>DLS 3:</u> 0123456789651230984782950473162304715698197062843556371849209846573102478196025330682915747459832061 (DLS *C*, CF 3);

<u>DLS 4:</u> 0123456789251096784338625940711034825967465173920897051826347986243510647930812552480713968397610452 (DLS *D*, CF 3);

<u>DLS 5:</u> 0123456789960728435154816720936095318427421976053829301458767852093164134682790587645392103578901642 (DLS *E*, CF 2);

<u>DLS 6:</u> 0123456789790138452685721906342354908167508761294396185374024260875391184906327567352498103496721058 (DLS *F*, CF 4);

<u>DLS 7:</u> 0123456789374516890256390872412381905467650783219479526418308064219375427639051898105746231498723056 (DLS *G*, CF 4).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

<u>CF 1:</u> 0123456789123409567823408195679876543210876590432164912780533508721946501763289476591804324982367105 (DLS *A*);

<u>CF 2:</u> 0123456789120453896743718092566948075321865294701375391624089810623574548679013227953816403067214895 (DLS *B*, DLS *E*);

<u>CF 3:</u> 0123456789123408957648519203675076831492249710365839806752149612748035634851792075093628418765294103 (DLS *C*, DLS *D*);

<u>CF 4:</u> 0123456789120453896724968051733865172094751238964059487613029630247851875192043663870942154079613528 (DLS *F*, DLS *G*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 7, a = 6, \rho = [1, 1, 1, 1, 1, 1, 6]$.

Method of finding:

Brute Force for horizontally symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

11. Seven (1:7 structure)

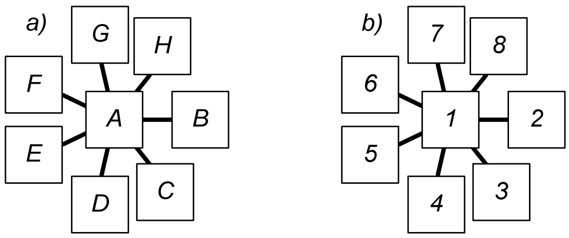

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 01234567891204379568394071582690318476525897620143756809243164751 38290261958430783529610744786203915 (DLS *A*, CF 1);

<u>DLS 2:</u> 01234567893967541802865907231424187059364572183690908623714572319 64058534069827117948205636805319427 (DLS *B*, CF 2);

<u>DLS 3:</u> 01234567893961542807805926731474186259301540783296968213407527349 01658537609842142978105636805379142 (DLS *C*, CF 3);

<u>DLS 4:</u> 01234567893961542807805296731474186952301540783926268913407597342 01658537602849142978105636805379142 (DLS *D*, CF 4);

<u>DLS 5:</u> 01234567893961542807805296731474186952301540783962628913407597342 01658537602849146978105232805379146 (DLS *E*, CF 5);

<u>DLS 6:</u> 01234567893961542807805926731474186259301540783296968213407527340 91658537690842142078195636895370142 (DLS *F*, CF 6);

<u>DLS 7:</u> 01234567893961542807865902731474182059361542783690908613427527349 61058537069842142978105636805379142 (DLS *G*, CF 7);

<u>DLS 8:</u> 01234567893961542807865209731474189052361549783620208613497597342 61058537062849142978105636805379142 (DLS *H*, CF 8).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

<u>CF 1:</u> 0123456789120437956839407158269031847652589762014375680924316475138 290261958430783529610744786203915 (DLS *A*);

<u>CF 2:</u> 0123456789123058964738927650144586271930741832059697046132585967804 321265914780363450981728071932465 (DLS *B*);

<u>CF 3:</u> 0123456789120459786394603281752839670514398674205187452196304698135 207657180439273520819465017963428 (DLS *C*);

<u>CF 4:</u> 0123456789123460985735879124062906341578841973526048502976139378564 021576208319460451789327691820345 (DLS *D*);

<u>CF 5:</u> 0123456789123708964547506913282346578091796482513090827134568591364 207680523791436189405725479102863 (DLS *E*);

CF 6: 0123456789120479836535978610429086347521795812463048305792165461203
978674201589323196804578675932104 (DLS *F*);
CF 7: 0123456789120458796394603251782839670514358674209157482196304695138
207697180435273520918468017963425 (DLS *G*);
CF 8: 0123456789120654897347380921655917364802865413902794726805313890275
416738592164065417032982069817354 (DLS *H*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 8, a = 7, \rho = [1, 1, 1, 1, 1, 1, 1, 7]$.

Method of finding:

search for partially horizontally symmetric DLSs, symmetric filled cells number $M = 60$, in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

12.1. Symmetric eight (1:8 structure)

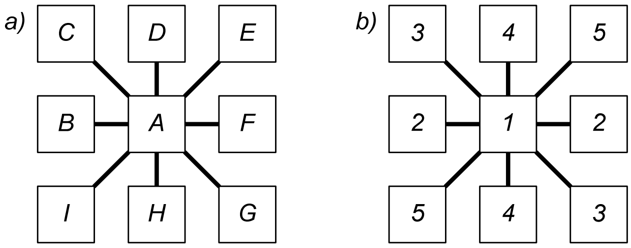

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 0123456789123054967849682713056874905213735918046294876321503591728046874609352150123678942605814937 (DLS *A*, CF 1) – horizontal symmetry;

DLS 2: 0123456789790136854234761892508532697401176053492828950436176258901374401927586396478120355384720196 (DLS *B*, CF 2);

DLS 3: 0123456789290136854734761892508532697401176053492878950436126258901374401972586396478120355384270196 (DLS *C*, CF 3);

DLS 4: 0123456789790136854234768192501532697408876053492128950436176258901374401927586396471820355384720196 (DLS *D*, CF 4);

DLS 5: 0123456789290136854734768192501532697408876053492178950436126258901374401972586396471820355384270196 (DLS *E*, CF 5);

DLS 6: 0123456789754136890294701832568952037641170564932828365940175268901473631427089546978125303089725164 (DLS *F*, CF 2);

DLS 7: 0123456789254136890794701832568952037641170564932878365940125268901473631472089546978125303089275164 (DLS *G*, CF 3);

DLS 8: 0123456789754136890294708132561952037648870564932128365940175268901473631427089546971825303089725164 (DLS *H*, CF 4);

DLS 9: 0123456789254136890794708132561952037648870564932178365940125268901473631472089546971825303089275164 (DLS *I*, CF 5).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789123054967849682713056874905213735918046294876321503591728046874609352150123678942605814937 (DLS *A*);

CF 2: 0123456789120436795889602753419837520614765918243024986315073582904176601574982357468130924371098265 (DLS *B*, DLS *F*);

CF 3: 0123456789120463795889302756419867520314735918246024983615073582904176601574982357468130924671098235 (DLS *C*, DLS *G*);

CF 4: 0123456789123409587664879012537605834921951278364047981620353850219 467896154730223496705185076328194 (DLS *D*, DLS *H*);
CF 5: 0123456789123409587665879012437605834921941278365047981620353840219 567896154730223596704185076328194 (DLS *E*, DLS *I*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 9, a = 8, \rho = [1, 1, 1, 1, 1, 1, 1, 1, 8].$$

Method of finding:

Brute Force for horizontally symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

12.2. Asymmetric eight (1:8 structure)

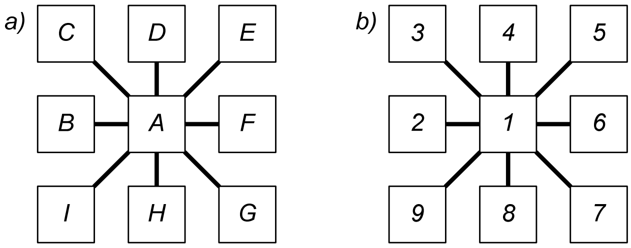

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 01234567891204573968573219480630189456728946710253485136209775608
2913493756084216489237510269708 1345 (DLS *A*, CF 1);

<u>DLS 2:</u> 01234567892385697041106483957295471803266418572930397021546842913
0865778390642155602741893875692 3104 (DLS *B*, CF 2);

<u>DLS 3:</u> 01234567892385967041106483957295471803266418572930397021546842913
0865778360942155602741893875962 3104 (DLS *C*, CF 3);

<u>DLS 4:</u> 01234567892385697014406183957295471803266418572930397024516812943
0865778390612455602714893875692 3401 (DLS *D*, CF 4);

<u>DLS 5:</u> 01234567892385967014406183957295471803266418572930397024516812943
0865778360912455602714893875962 3401 (DLS *E*, CF 5);

<u>DLS 6:</u> 01234567892385697041186403957295471803266410572938397821546042913
0865770398642155602741893875692 3104 (DLS *F*, CF 6);

<u>DLS 7:</u> 01234567892385967041186403957295471803266410572938397821546042913
0865770368942155602741893875962 3104 (DLS *G*, CF 7);

<u>DLS 8:</u> 01234567892385697014486103957295471803266410572938397824516012943
0865770398612455602714893875692 3401 (DLS *H*, CF 8);

<u>DLS 9:</u> 01234567892385967014486103957295471803266410572938397824516012943
0865770368912455602714893875962 3401 (DLS *I*, CF 9).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

<u>CF 1:</u> 0123456789120457396857321948063018945672894671025348513620977560829
134937560842164892375102697081345 (DLS *A*);

<u>CF 2:</u> 0123456789120463597878395406124391728065256890314769471825035612097
834875036942194862713503075814296 (DLS *B*);

<u>CF 3:</u> 0123456789120463597878395406124591728063238690154769471823055612097
834875036942194682731503075814296 (DLS *C*);

<u>CF 4:</u> 0123456789120463597839528674014319270865804572319674601893525678901 234983754261025963180476781094523 (DLS *D*);

<u>CF 5:</u> 0123456789120463597839528674014319270865806572319476401893525478901 236983754261025963180476781094523 (DLS *E*);

<u>CF 6:</u> 0123456789120463597828395406174391278065756890314269471825035612097 834875036942194867213503075814296 (DLS *F*);

<u>CF 7:</u> 0123456789120463597828395406174591278063738690154269471823055612097 834875036942194687231503075814296 (DLS *G*);

<u>CF 8:</u> 0123456789120436895798621753408347091562641952087370586342912670849 135398571240657962830144531907628 (DLS *H*);

<u>CF 9:</u> 0123456789120463597823469085176958721304549718206335192708467860349 152873509462196825174304071863295 (DLS *I*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 9, a = 8, \rho = [1, 1, 1, 1, 1, 1, 1, 1, 8]$.

Method of finding:

search for partially central symmetric DLSs, symmetric filled cells number $M = 60$, in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

13. Symmetric ten (1:10 structure)

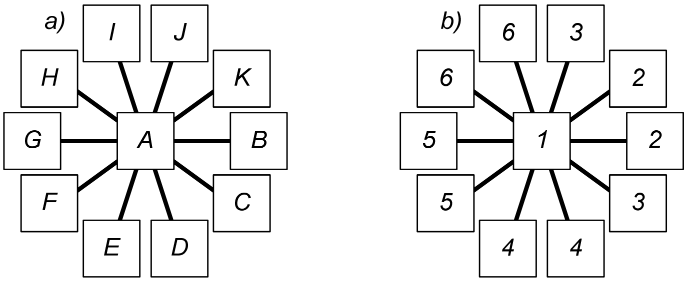

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 01234567891204879563754291830627810356948639507421945826103750976 43812631578294038601942754976320158 (DLS *A*, CF 1) – generalized symmetry (4,31,31);

<u>DLS 2:</u> 01234567893851694207549087263190162485734985713062163752049867023 81954854910732623789651407264039815 (DLS *B*, CF 2);

<u>DLS 3:</u> 01234567894851693207539087264190162485733985714062164752039867023 81954853910742624789651307264039815 (DLS *C*, CF 3);

<u>DLS 4:</u> 01234567893851694207549017263890162485734985713062863752049167023 81954154980732623789651407264039815 (DLS *D*, CF 4);

<u>DLS 5:</u> 01234567894851693207539017264890162485733985714062864752039167023 81954153980742624789651307264039815 (DLS *E*, CF 4);

<u>DLS 6:</u> 01234567893851694207549708263190162485734905713862163052749867823 01954854917032623789651407264839015 (DLS *F*, CF 5);

<u>DLS 7:</u> 01234567894851693207539708264190162485733905714862164052739867823 01954853917042624789651307264839015 (DLS *G*, CF 5);

<u>DLS 8:</u> 01234567893851694207549780263190162485734985713062163052749867023 81954854917032623789651407264039815 (DLS *H*, CF 6);

<u>DLS 9:</u> 01234567894851693207539780264190162485733985714062164052739867023 81954853917042624789651307264039815 (DLS *I*, CF 6);

<u>DLS 10:</u> 01234567893851694207549710263890162485734985713062863052749167023 81954154987032623789651407264039815 (DLS *J*, CF 3);

<u>DLS 11:</u> 01234567894851693207539710264890162485733985714062864052739167023 81954153987042624789651307264039815 (DLS *K*, CF 2).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

<u>CF 1:</u> 012345678912048795637542918306278103569486395074219458261037509764381263157829403860194275497632015 8 (DLS *A*);

<u>CF 2:</u> 01234567891204368957681270934534981725062650917438537984016247362850918947523610908563127475610948 23 (DLS *B,* DLS *K*);

<u>CF 3:</u> 01234567891204379568364918025740589376216385704192291756380498712450365736812940749062831585620914 73 (DLS *C,* DLS *J*);

<u>CF 4:</u> 01234567891204589367635729810479618352404598672031978234061554701638928645701923381902457620369174 58 (DLS *D,* DLS *E*);

<u>CF 5:</u> 01234567891204768935753984016256982743106815923074396701254890465378212350681497478210965384713952 06 (DLS *F,* DLS *G*);

<u>CF 6:</u> 01234567891204589367365709812479618352404598672031978234061554701638928345721906681920457320369174 58 (DLS *H*, DLS *I*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 11, a = 10, \rho = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 10]$.

Method of finding:

search for generalized symmetric DLSs with (4,31,31) generalized symmetry in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

14. Rhombus-3

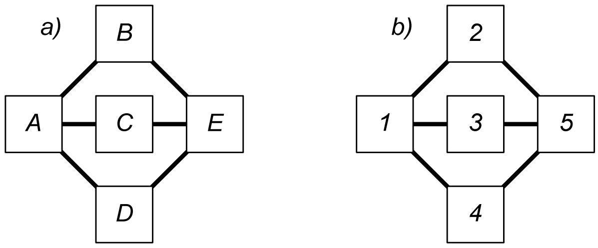

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 0123456789123704896560593124788904621537786193520436752840912490867153458617932097485036125312790846 (DLS *A*, CF 1);

<u>DLS 2:</u> 0123456789436029587118729603543541872906293768451050981276439615748032820951346774560312986784309125 (DLS *B*, CF 2);

<u>DLS 3:</u> 0123456789486029537113729608542541873906893762451050981376429615742038320951846774560812936784309125 (DLS *C*, CF 3);

<u>DLS 4:</u> 0123456789486029537113729608542541873906893764251050981376429615724038340951826772560814936784309125 (DLS *D*, CF 4);

<u>DLS 5:</u> 0123456789123704896560593124788904521637786193520435762840912490867153468517932097486035125312790846 (DLS *E*, CF 5).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

<u>CF 1:</u> 0123456789123704896560593124788904621537786193520436752840912490867153458617932097485036125312790846 (DLS *A*);

<u>CF 2:</u> 0123456789120467895369875304124831027695275936180456982430713570819246741698253093427051688065194327 (DLS *B*);

<u>CF 3:</u> 0123456789120458793629370156486895730412941862305753498721608761294503367014982575863012944052968371 (DLS *C*);

<u>CF 4:</u> 0123456789123406985759468170237308291546375968241080123759644897523601956074813264751302982681904375 (DLS *D*);

<u>CF 5:</u> 0123456789123679084567489321504815203697307182596493645712082587649031540916837276920845138950317426 (DLS *E*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 5, a = 6, \rho = [2, 2, 2, 3, 3]$.

Method of finding:

search for partially central symmetric DLSs, symmetric filled cells number $M = 60$, in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

15.1. Rhombus-4 (6 CF)

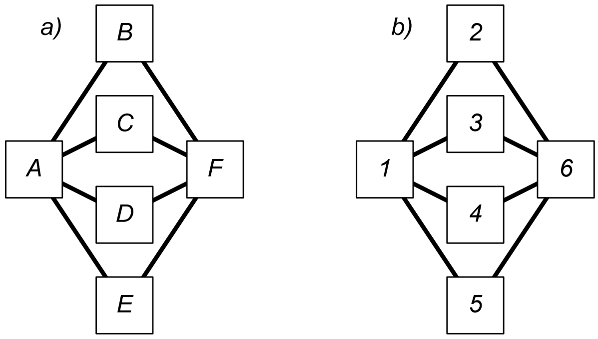

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 01234567891204368957941827536023579804168960527134584679120360718 43592378910462576950328414532619078 (DLS *A*, CF 1);

<u>DLS 2:</u> 01234567893769081245608594213746928753019548103672741063952859072 64813283451796012763980548351720496 (DLS *B*, CF 2);

<u>DLS 3:</u> 01234567893796081245608594213749628753019548103672741063952856072 94813283451796012793680548351720496 (DLS *C*, CF 3);

<u>DLS 4:</u> 01234567893769081245608594713246928753019548103627241063957859072 64813783451296012763980548351720496 (DLS *D*, CF 4);

<u>DLS 5:</u> 01234567893796081245608594713249628753019548103627241063957856072 94813783451296012793680548351720496 (DLS *E*, CF 5);

<u>DLS 6:</u> 01234567891204568937941827356023579804168960327154384679120560718 45392578910462376950328414532619078 (DLS *F*, CF 6).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

<u>CF 1:</u> 01234567891204368957941827536023579804168960527134584679120360718435 92378910462576950328414532619078 (DLS *A*);

<u>CF 2:</u> 01234567891204879536604853719297362458013591784260736901842584529630 17587062194326871903544915302678 (DLS *B*);

<u>CF 3:</u> 01234567891206798345506487319274519806239647532801873524901643981672 50387201596425896014376910324578 (DLS *C*);

<u>CF 4:</u> 01234567891234590678476290835183476210959071843562598621743076981352 04280536491735190728466450789123 (DLS *D*);

<u>CF 5:</u> 01234567891230579648305961487289743625014597108326246578019397862430 15681203795473418952605608921437 (DLS *E*);

<u>CF 6:</u> 01234567891204568937941827356023579804168960327154384679120560718453 92578910462376950328414532619078 (DLS *F*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 6, a = 8, \rho = [2, 2, 2, 2, 4, 4]$.

Method of finding:

search for partially central symmetric DLSs, symmetric filled cells number $M = 60$, in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

15.2. Rhombus-4 (5 CF)

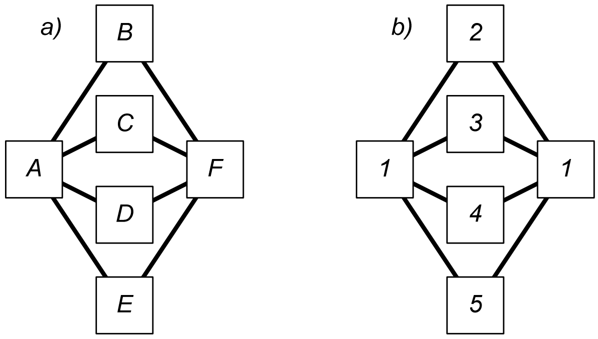

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 0123456789123409567834179028565976813402975836124045601893276805247913864972053170826341952391578064 (DLS *A*, CF 1);
- DLS 2: 0123456789374081952682640953716519270843730518496298375426102051638497147690325856987210344982367105 (DLS *B*, CF 2) – horizontal symmetry;
- DLS 3: 0123456789374081952682640953716519720843230518496798325476107051638492147690325856982710344987362105 (DLS *C*, CF 3) – horizontal symmetry;
- DLS 4: 0123456789375081942682640953716419270853730518496298375426102041638597157690324856987210344982367105 (DLS *D*, CF 4) – horizontal symmetry;
- DLS 5: 0123456789375081942682640953716419720853230518496798325476107041638592157690324856982710344987362105 (DLS *E*, CF 5) – horizontal symmetry;
- DLS 6: 0123456789123409567834179028567956813204957836142027601893456802574913864972053140856371925391248067 (DLS *F*, CF 1).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789123409567834179028565976813402975836124045601893276805247913864972053170826341952391578064 (DLS *A*, DLS *F*);
- CF 2: 0123456789120478963595703421682915634870569280734170382659148347091256375691840248615230976489170523 (DLS *B*);
- CF 3: 0123456789120436597837850941265379810264964172853028605493177958631402403218769584962730516517902843 (DLS *C*);
- CF 4: 0123456789120478963595703481628915634270569280734170382659142347091856375691240848615230976489170523 (DLS *D*);
- CF 5: 0123456789120463597837850941265679810234934172856028605493177958361402403218769584962730516517902843 (DLS *E*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 6, a = 8, \rho = [2, 2, 2, 2, 4, 4]$.

Method of finding:

Brute Force for horizontally symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

15.3. Rhombus-4 (4 CF)

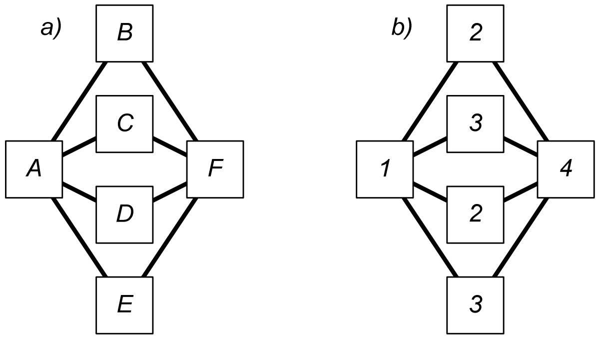

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 0123456789120463597837105498262539810647589672301440873621957641908532935827146084620973516975184203 (DLS *A*, CF 1) – horizontal symmetry;

<u>DLS 2:</u> 0123456789984536712085796304127956123804438190256712607493586798215043301758429626048719355432098671 (DLS *B*, CF 2);

<u>DLS 3:</u> 0123456789984536712085796304127056123894438190256712607493586708215943391758420626948710355432098671 (DLS *C*, CF 3);

<u>DLS 4:</u> 0123456789978236451078596302415916783402234790816514605293786594871023307514289646082159378231097654 (DLS *D*, CF 2);

<u>DLS 5:</u> 0123456789978236451078596302415016783492234790816514605293786504871923397514280646982150378231097654 (DLS *E*, CF 3);

<u>DLS 6:</u> 0123456789129463507837105498262539810647580672391449873621057641098532935827146084629073516075184293 (DLS *F*, CF 4) – horizontal symmetry.

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

<u>CF 1:</u> 0123456789120463597837105498262539810647589672301440873621957641908532935827146084620973516975184203 (DLS *A*);

<u>CF 2:</u> 0123456789120457396854807391268549260371706834159297368124503895627014497218560363510982472617904835 (DLS *B*, DLS *D*);

<u>CF 3:</u> 0123456789120436597823159048673469780152789154203656801793249742638510493782160585760932416058217493 (DLS *C*, DLS *E*);

<u>CF 4:</u> 0123456789120463597829315486074698271035948672315065798102433750189426804236759173150948625867902314 (DLS *F*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 6, a = 8, \rho = [2, 2, 2, 2, 4, 4]$.

Method of finding:

Brute Force for horizontally symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

16.1. Symmetric fish

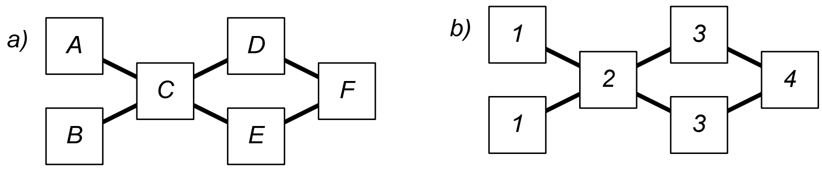

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 0123456789120436795887965240316945182370451073982620578136949371608542386927041554380912677682945103 (DLS *A*, CF 1);

DLS 2: 0123456789473190862524196308577894365012805627349169751842035260819374134209756896875421303508721946 (DLS *C*, CF 2) – horizontal symmetry;

DLS 3: 0123456789120436795887965240319645182370451073982620578136946371908542386927041554380912677982645103 (DLS *D*, CF 3);

DLS 4: 0123456789140236597886957430219267184503371062984550368124977541938260485927031623780916546984507132 (DLS *B*, CF 1);

DLS 5: 0123456789140236597886957430219267184530371062984550368124977541908263485927031623780916546984537102 (DLS *E*, CF 3);

DLS 6: 0123456789473190862574196308522894365017805627349169751842035260819374134709256896825471303508721946 (DLS *F*, CF 4) – horizontal symmetry.

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120436795887965240316945182370451073982620578136949371608542386927041554380912677682945103 (DLS *A*, DLS *B*);

CF 2: 0123456789123054967846789012355961278304875936042175961830422805634917934781256060147258933482097156 (DLS *C*);

CF 3: 0123456789120436795887965240319645182370451073982620578136946371908542386927041554380912677982645103 (DLS *D*, DLS *E*);

CF 4: 0123456789120487963589367145022579108346908562147376580432914897365120571093286434625970186341280957 (DLS *F*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 6, a = 6, \rho = [1, 1, 2, 2, 2, 4]$.

Method of finding:

Brute Force for horizontally symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

16.2. Asymmetric fish

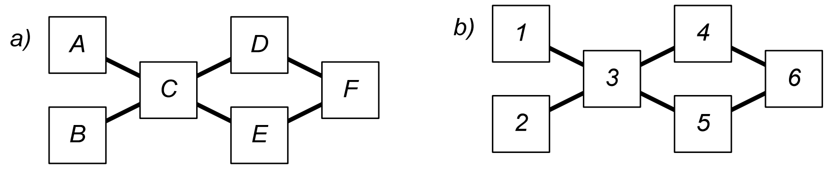

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 0123456789120436795849125780365679143802375861429073912806452037895164958603142784659023716840729513 (DLS *A*, CF 1);

DLS 2: 0123456789945702861352619043784018392567837654190225408791366795213840380916725416327804957984635021 (DLS *C*, CF 2);

DLS 3: 0123456789120536794849125780365679143802374861529073912806542037894165948603152785649023716850729413 (DLS *D*, CF 3);

DLS 4: 0123456789120536794849825710365679143802374168529073982106542037894165941603852785649023716850729413 (DLS *E*, CF 4);

DLS 5: 0123456789120436795849825710365679143802375168429073982106452037895164951603842784659023716840729513 (DLS *B*, CF 5);

DLS 6: 0123456789945702861352691043784018392567837654910225408719366795213840380196725416327804957984635021 (DLS *F*, CF 6).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120458396763890475128975612340469713025827368954019860374125501872963434529018767541268093 (DLS *A*);

CF 2: 0123456789120437985658917406232476108395963581740240872359613950681247674209351883695241707518962034 (DLS *B*);

CF 3: 0123456789120436795849125780365679143802375861429073912806452037895164958603142784659023716840729513 (DLS *C*);

CF 4: 0123456789120463795837819602452059783164957210843659368410274697315802731852469084650923716840279513 (DLS *D*);

CF 5: 0123456789120436795849825710365679143802375168429073982106452037895164951603842784659023716840729513 (DLS *E*);

CF 6: 0123456789120458396763890475128975612340269713045847368952019860374125501872963434529018767541268093 (DLS *F*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 6, a = 6, \rho = [1, 1, 2, 2, 2, 4]$.

Method of finding:

search for partially generalized symmetric DLSs with (4,31,31) symmetry, symmetric filled cells number $M = 70$, in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

17. Cross

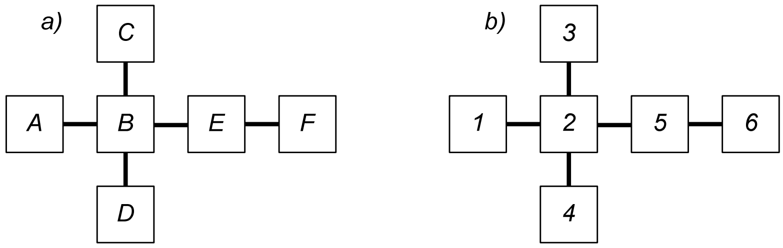

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 01234567891204678953754201369837682915044079385261938576041268105
29347593684712084519320762697104835 (DLS *A*, CF 1);

<u>DLS 2:</u> 01234567894638597012527916083410958724636842903175871634529029847
31506946021835735076849217351029648 (DLS *B*, CF 2);

<u>DLS 3:</u> 01234567898204671953754208369137612985044079315268938576041268105
29347593614782014589320762697804135 (DLS *C*, CF 3);

<u>DLS 4:</u> 01234567898207641953754208369137612985044079315268938576041268105
29347593617482014589320762694807135 (DLS *E*, CF 4);

<u>DLS 5:</u> 01234567891207648953754201369837682915044079385261938576041268105
29347593687412084519320762694107835 (DLS *D*, CF 5);

<u>DLS 6:</u> 01234567894618597032527936081410958724636842901375873614529029847
13506946023815735076849217351029648 (DLS *F*, CF 6).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

<u>CF 1:</u> 0123456789120467895375420136983768291504407938526193857604126810529
347593684712084519320762697104835 (DLS *A*);

<u>CF 2:</u> 0123456789123057986479623041582749680531630184592745172380968095761
342567809241398541236703486917205 (DLS *B*);

<u>CF 3:</u> 0123456789120657983437149856208965037142639784250145821903679648703
215785026149350316249782479318056 (DLS *C*);

<u>CF 4:</u> 0123456789120479865346598273106548371092901268354757962348013870169
425296754013884319052767385012964 (DLS *E*);

<u>CF 5:</u> 0123456789120459867396582104373417902865256983701487326495016841073
952798536412040761253985390781246 (DLS *D*);

<u>CF 6:</u> 0123456789123408957635816724902679540318409872563159071648237415893
062986230715463502189478746931205 (DLS *E*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 6, a = 5, \rho = [1, 1, 1, 1, 2, 4]$.

Method of finding:

search for partially central symmetric DLSs, symmetric filled cells number $M = 60$, in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

18. Flyer

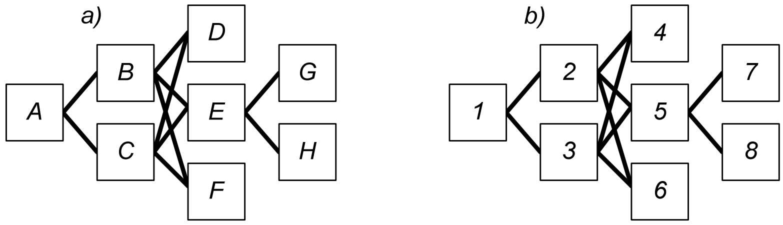

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 0123456789120463895740912853766947501823735984016258367124908570963214978532460136120975482468179035 (DLS *A*, CF 1);

DLS 2: 0123456789438709261516598734029234618570657890423120153698473492187056786024519387065319245941720368 (DLS *B*, CF 2);

DLS 3: 0123456789483709261516593784029284613570657890423120158693473492187056736024519887065319245941720863 (DLS *C*, CF 3);

DLS 4: 0123456789120463895740912853766917504823735981046258367421908570963241978532160436420975182468179035 (DLS *D*, CF 4);

DLS 5: 0123456789120463895740912853766947521803735984216058367104928570963214978530462136120975482468179035 (DLS *E*, CF 5);

DLS 6: 0123456789120463895740912853766917524803735981246058367401928570963241978530162436420975182468179035 (DLS *F*, CF 6);

DLS 7: 0123456789438709261516508734929234618570657890423120153698473402187956786924510387965310245941720368 (DLS *G*, CF 7);

DLS 8: 0123456789483709261516503784929284613570657890423120158693473402187956736924510887965310245941720863 (DLS *H*, CF 8).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120463895740912853766947501823735984016258367124908570963214978532460136120975482468179035 (DLS *A*);

CF 2: 0123456789120458397626579148039748631520806534719243912780655470169238793280564135860924176819720354 (DLS *B*);

CF 3: 0123456789120458397626579418039718634520806531749243912780655470169238793280561435860921476849720351 (DLS *C*);

CF 4: 0123456789120463895740912853766917504823735981046258367421908570963241978532160436420975182468179035 (DLS *D*);

CF 5: 0123456789120463895740912853766947521803735984216058367104928570963214978530462136120975482468179035 (DLS *E*);

<u>CF 6:</u> 0123456789120463895740912853766917524803735981246058367401928570963241978530162436420975182468179035 (DLS *F*);
<u>CF 7:</u> 0123456789120657943834672158907695834021954812760348326901752970348516801976235467510839425384901267 (DLS *G*);
<u>CF 8:</u> 0123456789120657943834672158907685934021854912760348326901752970348516901876235467510839425394801267 (DLS *H*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 8, a = 10, \rho = [1, 1, 2, 2, 2, 4, 4, 4]$.

Method of finding:

search for partially central symmetric DLSs, symmetric filled cells number $M = 60$, in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

20. Tree-1

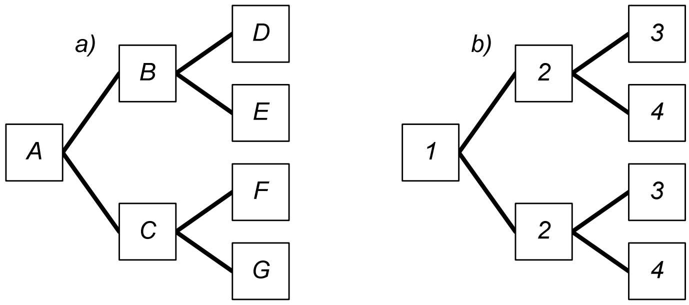

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 0123456789120467985384759106327061238594983652140737920841656547893210261034597853897620414958107326 (DLS *A*, CF 1) – generalized symmetry (4,31,31);

DLS 2: 0123456789637589204110496253782908147653458270391658372614907261389504945607813236905148278714930265 (DLS *B*, CF 2);

DLS 3: 0123456789647589203110396254782908147653358270491658472613907261389504935607814246905138278714930265 (DLS *C*, CF 2);

DLS 4: 0123456789120467985384759126307061238594983652140737920841656547893012261034597853897602414958107326 (DLS *D*, CF 3);

DLS 5: 0123456789123467985084759126037061238594980652143737920841656547893012261034597853897602414958107326 (DLS *E*, CF 4);

DLS 6: 0123456789120467985354789106327061238594983652140737920841656547893210261034597883597620414985107326 (DLS *F*, CF 3);

DLS 7: 0123456789120867945358749106327061238594943652180737920841656547893210261034597883597620414985107326 (DLS *G*, CF 4).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120467985384759106327061238594983652140737920841656547893210261034597853897620414958107326 (DLS *A*);

CF 2: 0123456789123406985774581906325796843210381297540646072813959381527064654930217820756389418960714523 (DLS *B*, DLS *C*);

CF 3: 0123456789120458793656489302176459108372936074152887920136454876325190391567280425378940617081269453 (DLS *D*, DLS *F*);

CF 4: 0123456789120458793656489302176459108372236074159887920136454876395120391567280495378240617081269453 (DLS *E*, DLS *G*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 7, a = 6, \rho = [1, 1, 1, 1, 2, 3, 3]$.

Method of finding:

search for generalized symmetric DLSs with (4,31,31) generalized symmetry in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

Remark:

for different orders *N* of DLSs it is well known different kinds of trees.

21. Venus

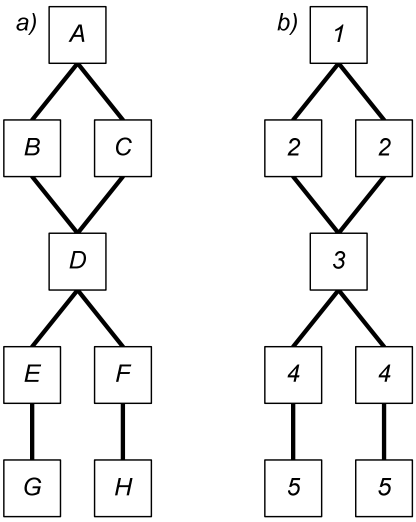

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 0123456789120478963573856014925916837204365291804790785423164590273168846719052368413259702739064851 (DLS *A*, CF 1) – generalized symmetry (4,31,31);

<u>DLS 2:</u> 0123456789589621734030485691271672098435876012359442593716082437905816730168495295847320616915840273 (DLS *B*, CF 2);

<u>DLS 3:</u> 0123456789589621734039485601271672908435876912350442503716982437095816730168495295847320616015849273 (DLS *C*, CF 2);

<u>DLS 4:</u> 0123456789120476983573658014925918237064385691024790725483164590673128648719250386413259702739084651 (DLS *D*, CF 3) – generalized symmetry (4,31,31);

<u>DLS 5:</u> 0123456789859621734030485691271672095438576012389442893716052437908516730168495298547320616915840273 (DLS *E*, CF 4);

<u>DLS 6:</u> 0123456789859621734039485601271672905438576912380442803716952437098516730168495298547320616015849273 (DLS *F*, CF 4);

<u>DLS 7:</u> 0123456789127406983573658014925918237064385691024797025483164590673128648719250386413259702039784651 (DLS *G*, CF 5);

<u>DLS 8:</u> 0123456789120476983578653014925918237064835691024790725483164590673128648719250336418259702739084651 (DLS *H*, CF 5).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120478963573856014925916837204365291804790785423164590273
1688467190523684132597027390648551 (DLS *A*);
CF 2: 0123456789120459863730578194269546327801591870236446701832956495231
0788361074952783296514027896405513 (DLS *B*, DLS *C*);
CF 3: 0123456789120476983573658014925918237064385691024790725483164590673
1286487192503864132597027390846551 (DLS *D*);
CF 4: 0123456789120459863730678194259645327801691870235445701832965496231
0788351074962783296514027896405513 (DLS *E*, DLS *F*);
CF 5: 0123456789120476983578653014925918237064835691024790725483164590673
1286487192503364182597027390846551 (DLS *G*, DLS *H*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 8, a = 8, \rho = [1, 1, 2, 2, 2, 2, 2, 4]$.

Method of finding:

search for generalized symmetric DLSs with (4,31,31) generalized symmetry in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

22. Daedalus-8

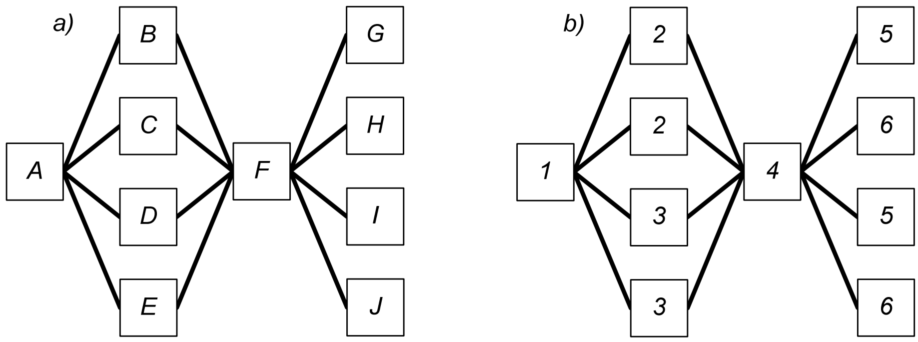

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 0123456789150679342838640251975917384602405283791627915408637280619354637910824596482715308435962071 (DLS *A*, CF 1) – generalized symmetry (4,31,31);

<u>DLS 2:</u> 0123456789534017986264759182307596803124961854207339840216578037264591425138790628697304151702695348 (DLS *B*, CF 2);

<u>DLS 3:</u> 0123456789534017986264759812307596803124968154207339140286578037264591425831790628697304151702695348 (DLS *C*, CF 2);

<u>DLS 4:</u> 0123456789534917086264759182307506893124961854207339840216578037264591425138790628607394151792605348 (DLS *D*, CF 3);

<u>DLS 5:</u> 0123456789534917086264759812307506893124968154207339140286578037264591425831790628607394151792605348 (DLS *E*, CF 3);

<u>DLS 6:</u> 0123456789120679345838640251972917384605405283791657912408637580619324637910854296485712308435962071 (DLS *F*, CF 4) – generalized symmetry (4,31,31);

<u>DLS 7:</u> 0123456789534017986284759612307598603124968154207339140286576037284591425631790828697304151702895346 (DLS *G*, CF 5);

<u>DLS 8:</u> 0123456789534917086284759612307508693124968154207339140286576037284591425631790828607394151792805346 (DLS *H*, CF 6);

<u>DLS 9:</u> 0123456789534017986264759182307596801324963854207119840236578017264593425138790628697304153702695148 (DLS *I*, CF 5);

<u>DLS 10:</u> 0123456789534917086264759182307506891324963854207119840236578017264593425138790628607394153792605148 (DLS *J*, CF 6).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

<u>CF 1:</u> 0123456789123978460595672104385314602897270896531470825391464691378 052847619352068450219733950847261 (DLS *A*);

<u>CF 2:</u> 0123456789120458936785926170349658734120246037895159762408137035891 642431706259868491032753781925406 (DLS *B*, DLS *C*);

<u>CF 3:</u> 0123456789120437956823681974509475082136658973104239428156078731604 925465092387150172683947896540213 (DLS *D*, DLS *E*);

<u>CF 4:</u> 0123456789120679345838640251972917384605405283791657912408637580619 324637910854296485712308435962071 (DLS *F*);

<u>CF 5:</u> 0123456789120458936789526170345698734120246037859195762408137035891 642431706295868491032753781925406 (DLS *G*, DLS *I*);

<u>CF 6:</u> 0123456789120437965824518973605689714032937062814587451309264938261 507356704289160129854737896503214 (DLS *H*, DLS *J*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 10, a = 12, \rho = [1, 1, 1, 1, 2, 2, 2, 2, 4, 8]$.

Method of finding:

search for generalized symmetric DLSs with (4,31,31) generalized symmetry in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

22. Daedalus-10

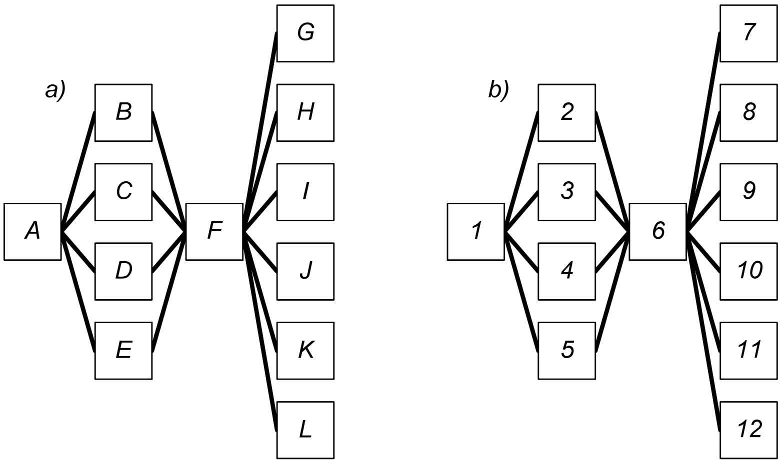

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

<u>DLS 1:</u> 01234567891204537968946238517045186920373986741205275981034678302 69514867590342163910748525047128693 (DLS *A*, CF 1);

<u>DLS 2:</u> 01234567898756092341639487105292457138602430185976598726410330716 28495486953021715029476387618309524 (DLS *B*, CF 2);

<u>DLS 3:</u> 01234567898756092341639417805292457831602430815976591726480330786 21495486953021715029476387681309524 (DLS *C*, CF 3);

<u>DLS 4:</u> 01234567898756092341239487105696457138206430185972598726410330716 28495486953021715029476387218309564 (DLS *D*, CF 4);

<u>DLS 5:</u> 01234567898756092341239417805696457831206430815972591726480330786 21495486953021715029476387281309564 (DLS *E*, CF 5);

<u>DLS 6:</u> 01234567891204537968976238514075186920343986741205245981037648302 69517867590342163910748525047128693 (DLS *F*, CF 6);

<u>DLS 7:</u> 01234567898756092341639417805292457031682430815976591726480330786 21495486953021715829476307601389524 (DLS *G*, CF 7);

<u>DLS 8:</u> 01234567898756092143639417805292457013682410835976593726480110786 23495486951023735829476107601389524 (DLS *H*, CF 8);

<u>DLS 9:</u> 01234567898756092143639417805292457813602410835976593726480110786 23495486951023735029476187681309524 (DLS *I*, CF 9);

<u>DLS 10:</u> 01234567898756092341239417805696457031286430815972591726480330786 21495486953021715829476307201389564 (DLS *J*, CF 10);

<u>DLS 11:</u> 01234567898756092143239417805696457013286410835972593726480110786 23495486951023735829476107201389564 (DLS *K*, CF 11);

<u>DLS 12:</u> 01234567898756092143239417805696457813206410835972593726480110786 23495486951023735029476187281309564 (DLS *L*, CF 12).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

<u>CF 1:</u> 0123456789120453796894623851704518692037398674120527598103467830269514867590342163910748525047128693 (DLS *A*);

<u>CF 2:</u> 0123456789120436597839568174024619270835806572319453421890672478901356983754261075906382416781094523 (DLS *B*);

<u>CF 3:</u> 0123456789120457396894867251033759168420269730485175382410968965017234507183964263409825174812690375 (DLS *C*);

<u>CF 4:</u> 0123456789120436597823586914077869530142641072985340928173653541078296873590462196871425305976283014 (DLS *D*);

<u>CF 5:</u> 0123456789120459763847869301529035642817657138942038927610457650814293236817590484190235765947208361 (DLS *E*);

<u>CF 6:</u> 0123456789120453796897623851407518692034398674120524598103764830269517867590342163910748525047128693 (DLS *F*);

<u>CF 7:</u> 0123456789120463597878395146026091728543254890316743671802955612097834875036942194862713503975842016 (DLS *G*);

<u>CF 8:</u> 0123456789120459863775129034689386015274463872195027591803463960274815847136950258476320916095847123 (DLS *H*);

<u>CF 9:</u> 0123456789120463597839568174024319270865804572319654621890372678901354983754261075903682416781094523 (DLS *I*);

<u>CF 10:</u> 0123456789120463597828395146076091278543754890316243671802955612097834875036942194867213503975842016 (DLS *J*);

<u>CF 11:</u> 0123456789120459863775123094683986015274463872195027591803469360274815847196350258476320916095847123 (DLS *K*);

<u>CF 12:</u> 0123456789120436895798621753405347981602641952087370586342912670849135398501742687962035144531792068 (DLS *L*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 12,\ a = 14,\ \rho = [1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 4, 10]$.

Method of finding:

search for partially central symmetric DLSs, symmetric filled cells number $M = 60$, in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

23. Robot

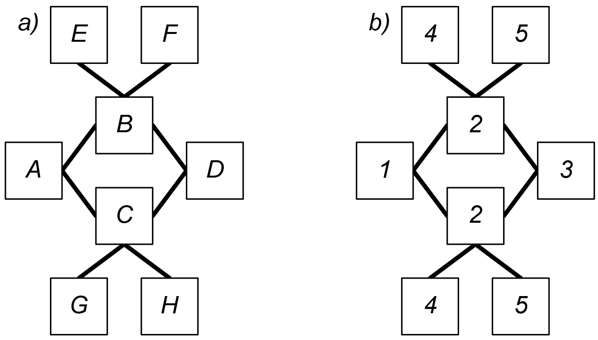

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 01234567891204785963238160459745391620787460519832967503821450983 4762169428731508716290345385792140 6 (DLS *A*, CF 1) – generalized symmetry (4,31,31);

DLS 2: 01234567898310967524489672301572645819301958642307604137589295072 1846337851092462679034158543289067 1 (DLS *B*, CF 2);

DLS 3: 01234567898319067524489672301572645819301058642397694137580295072 18463378519024626709341585432809671 (DLS *C*, CF 2);

DLS 4: 01234567891204795863238160459745391620787460518932867503921450983 47621694287315097162803453857921406 (DLS *D*, CF 3) – generalized symmetry (4,31,31);

DLS 5: 01234567891234785960238160459745091623787460519832967503821450983 47621694287015387162930453857921406 (DLS *E*, CF 4);

DLS 6: 01234567891234795860238160459745091623787460518932867503921450983 47621694287015397162830453857921406 (DLS *F*, CF 5);

DLS 7: 01234567895204781963238160459745391620787460519832967503821410983 47625694287315087562903413817925406 (DLS *G*, CF 4);

DLS 8: 01234567895204791863238160459745391620787460518932867503921410983 47625694287315097562803413817925406 (DLS *H*, CF 5).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120478596323816045974539162078746051983296750382145098347 6216942873150871629034538579214 06 (DLS *A*);

CF 2: 0123456789120657893494578326016371940528859471326036102894572968304 1757849065312478562109350321978 46 (DLS *B*, DLS *C*);

CF 3: 0123456789120479586323816045974539162078746051893286750392145098347 6216942873150971628034538579214O6 (DLS D);

CF 4: 0123456789123457890654926318709756803142308174926563401825978917365 0247865920413450921763826780943 51 (DLS E, DLS G);

CF 5: 0123456789123459786056108793247346125098685930417285972406139471638 2053085762941270891345649620815 37 (DLS F, DLS H).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 8, a = 8, \rho = [1, 1, 1, 1, 2, 2, 4, 4]$.

Method of finding:

search for generalized symmetric DLSs with (4,31,31) generalized symmetry in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

24. Stingray

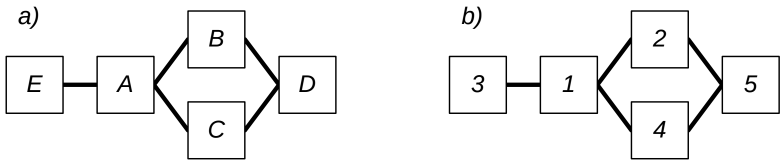

Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 0123456789120467893558367201943785942610291086534763410975284597281063765831940294625038718079134256 (DLS *A*, CF 1);

DLS 2: 0123456789549278016363719042582619873045926754183089501324761705628394358409762140382659177846319502 (DLS *B*, CF 2);

DLS 3: 0123456789541078926363712940582609871345306754289189523104761735628904958413762042980651377846903512 (DLS *E*, CF 3);

DLS 4: 0123456789541278906363710942582609871345326754089189503124761735628904958413762040982651377846903512 (DLS *C*, CF 4);

DLS 5: 0123456789320467891558167203941785942630293086514763410975284597283061765831940294625018738079134256 (DLS *D*, CF 5).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120467893558367201943785942610291086534763410975284597281063765831940294625038718079134256 (DLS *A*);

CF 2: 0123456789123486905760475928138659730124539017864229056843719582317460471602359874682019353871945206 (DLS *B*);

CF 3: 0123456789120458963737810245964675931208253867091473608954219452317860894620317550197683426897142053 (DLS *E*);

CF 4: 0123456789120458936767850214939547612830246837091576308945214371965208895620317450197386423892147056 (DLS *C*);

CF 5: 0123456789120467985359807236146795140238985631742070198653423567284901463809217584725310962341908567 (DLS *D*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$v = 5, a = 5, \rho = [1, 2, 2, 2, 3]$.

Method of finding:

search for partially central symmetric DLSs, symmetric filled cells number $M = 80$, in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

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