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**LIST OF THE COMBINATORIAL STRUCTURES FROM DLSS OF ORDER 10 ON
ORTHOGONALITY RELATIONSHIP SET**

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Basic definitions

DLS – diagonal Latin square (in this document of order 10).

Orthogonal DLSs (ODLS) – pair of DLS A and B , in which all ordered pairs of elements (a_{ij}, b_{ij}) are distinct.

String representation of DLS – elements of DLS that are written from left to right from top to bottom:

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3 2 8 4 6 7 1 0 9 5
8 1 2 7 4 6 3 5 0 9
1 5 0 9 8 2 4 3 7 6
6 8 5 2 0 9 7 1 4 3
9 0 7 1 5 4 2 6 3 8
4 3 9 0 1 8 6 7 5 2
0 6 3 8 7 5 9 2 1 4
5 7 6 3 9 1 8 4 2 0
7 9 4 5 2 3 0 8 6 1
2 4 1 6 3 0 5 9 8 7
```

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3284671095
8127463509
1509824376
6852097143
9071542638
4390186752
0638759214
5763918420
7945230861
2416305987
```

DLX – Dancing Links X algorithm – algorithm for exact cover problem solving.

Canonical form (CF) – lexicographically minimal string representation of DLS within corresponding isomorphism (isotopism) class.

Symmetry – some correspondence between elements of DLS. Can be viewed geometrically (vertical or horizontal plane symmetry, central symmetry) or generalized through structure of lengths of loops of permutations (P_x, P_y, P_v) . For $N = 10$ well known $10! = 3\,628\,800$ different permutations but only 42 from them has different structures of loops.

Partially symmetric DLS – DLS, in which only M cells corresponds to some symmetry and another cells has asymmetric filling, in correspondence with definition of correct DLS.

Combinatorial structure – graph, in which vertices are DLSs and edges are corresponds to the orthogonality binary relationship between pair of DLSs. The set of different combinatorial structures that are found within distributed computing project Gerasim@Home is shown below.

1. DLS without ODLS (bachelor)



Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 01234567891204539867356187409294783216052730985146685924731046970
13258704619852383156029745982760431 (DLS A , CF 1).

Adjacency matrix:

$$M = (0).$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120453986735618740929478321605273098514668592473104697013
258704619852383156029745982760431 (DLS A).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 1, a = 0, \rho = [1].$$

Method of finding:

any type of search due to most amount of DLSs hasn't ODLS.

2.1. ODLS pair without symmetry (asymmetric once)



Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 01234567891204365978201563489734579816205749802163698124730583760
19254963072854175681904324892573016 (DLS *A*, CF 1);

DLS 2: 01234567893758692410498710325612097458639412580637209436857168302
71945854601739256719340287365829104 (DLS *B*, CF 2).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120436597820156348973457981620574980216369812473058376019
254963072854175681904324892573016 (DLS *A*);

CF 2: 0123456789123067984589740613523759248016456812390776025841939847315
260241593067850867924316391807524 (DLS *B*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 2, a = 1, \rho = [1, 1].$$

Method of finding:

any type of search in combination with method of checking DLS for ODLS (Euler-Parker method, DLX [9]), approximately among 30 millions of DLS exists only 1 DLS with ODLS.

2.2. ODLS pair with symmetry (symmetric once)



Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 01234567891204587936543789162067981302549375628041894071356236512
49807708296541325693041784816072395 (DLS *A*, CF 1);

DLS 2: 01234567899357860421256107489346092175383786542910729438510689457
01362581063927410789236456432198057 (DLS *B*, CF 1).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120458793654378916206798130254937562804189407135623651249
807708296541325693041784816072395 (DLS *A*, DLS *B*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 2, a = 1, \rho = [1, 1].$$

Method of finding:

Brute Force for self-orthogonal ODLSs (SODLS) [1, 2] (symmetry provided by DLS transposing from the main diagonal or main anti-diagonal); also found different ODLS with same property for different M -transformations [3].

3.1. Line-3 without symmetry (asymmetric twice)



Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 01234567891204369857278693510480571923463869571420597104863276482
13095453062791894157802636392804571 (DLS *B*, CF 1);
DLS 2: 01234567899652017438527834901638195246071905268374603478259143918
75260748693012585671039422740691853 (DLS *A*, CF 2);
DLS 3: 01234567899652817430527034981638195246071985260374603478259143910
75268740693812585671039422748691053 (DLS *C*, CF 3).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120436985727869351048057192346386957142059710486327648213
095453062791894157802636392804571 (DLS *B*);
CF 2: 0123456789120478935643768012955089217643251764893076981354023960574
821873192056498453620176452093178 (DLS *A*);
CF 3: 0123456789120478935643798012655086217943251794863076981354023960574
821873162059498453620176452093178 (DLS *C*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 3, a = 2, \rho = [1, 1, 2].$$

Method of finding:

any type of search in combination with method of checking DLS for ODLS (Euler-Parker method, DLX), approximately for 500–1000 times it is present 1 asymmetric twice.

Remark:

line-3 structure also can be treated as «1:2 structure» or as rhombus-1.

3.2. Line-3 with symmetry (symmetric twice)



Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 01234567891204365978203581469746971820359786543120347809125669417
28503785063941253192708648562907341 (DLS B, CF 1) – horizontal
symmetry;

DLS 2: 01234567893857290416479836152092865031746510724893564281903783049
75261246918730510756329487931048652 (DLS A, CF 2);

DLS 3: 01234567893859072416974836102552869431706015729843269081753483742
05961496218035715076342987431598602 (DLS C, CF 2).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120436597820358146974697182035978654312034780912566941728
503785063941253192708648562907341 (DLS B);

CF 2: 0123456789120436895757169038423897524016645981027393782415608560179
324493278560126450971387081632495 (DLS A, DLS C).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 3, a = 2, \rho = [1, 1, 2].$$

Method of finding:

any type of symmetric DLS search (plane symmetry [4–6], generalized symmetry [7]) in
combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

4.1. Line-4 without symmetry



Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 01234567891204579836546820319726751389406789324051381079526479460
81325935786041280916425734532917608 (DLS *B*, CF 1);

DLS 2: 01234567897315948620365978401242986015739872163405508637294114072
39856694051723825348901678761025394 (DLS *A*, CF 2);

DLS 3: 01234567897315984620365974801242986015739472163805508637294118072
39456694051723825348901678761025394 (DLS *C*, CF 3);

DLS 4: 01234567891204539876746820159326153789406589724031387019526459460
83127973186045280976423154352917608 (DLS *D*, CF 4).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120457983654682031972675138940678932405138107952647946081
325935786041280916425734532917608 (DLS *B*);

CF 2: 0123456789123069584760578431925419082376284170963579683105244796238
051937256140835841279608605974213 (DLS *A*);

CF 3: 0123456789120456893735719248069486307152593281067487690423156895173
420704829156346107352982357689041 (DLS *C*);

CF 4: 0123456789120453896739872015468015947623749261503858760932146538724
190964187230543501698722769380451 (DLS *D*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 4, a = 3, \rho = [1, 1, 2, 2].$$

Method of finding:

search for partially generalized symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

4.2. Line-4 with symmetry



Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 01234567891238670945796408351246197028533802519467859734120624508
67391908623517457419286306375194028 (DLS A, CF 1);

DLS 2: 01234567897692148350143679082590853276145764231098397068514283415
79206250891346762198045734857062931 (DLS B, CF 2) – SODLS;

DLS 3: 01234567891238690475976408351246197028533802514967859734120629508
67341708623519454719286306345179028 (DLS C, CF 2) – SODLS;

DLS 4: 01234567897659148320143672089520853976149764231058397068514283415
79206520891346765128049734897062531 (DLS D, CF 1).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120453897626897051437836124095675098132484950176324972863
501356827941093176402585041392867 (DLS B, DLS C);

CF 2: 0123456789123867094579640835124619702853380251946785973412062450867
391908623517457419286306375194028 (DLS A, DLS D).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 4, a = 3, \rho = [1, 1, 2, 2].$$

Method of finding:

Brute Force of the SODLS.

5. Line-5 with symmetry



Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 01234567891204365978234918056787659043219482637150583027961476985
41032351709284640768132956951728403 (DLS C, CF 3) – horizontal symmetry;

DLS 2: 01234567893578092146726590431840812376956934785201875912046393426
18570140687395226105498375897361024 (DLS D, CF 2);

DLS 3: 01234567893587091246186590437240326781958974125603635978042192418
37560740621395826105498375798362014 (DLS B, CF 2);

DLS 4: 01234567891204365978234918056787659043219472638150583027961476985
41032351709284640867132956951827403 (DLS E, CF 1);

DLS 5: 01234567891204365978234918056787659043219481637250583027961476985
41032351709284640768231956952718403 (DLS A, CF 1).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120436597823491805678765904321947263815058302796147698541
032351709284640867132956951827403 (DLS A, DLS E);

CF 2: 0123456789120479563834968120576075184392285736941085602739414638901
275731954082697410285635982637104 (DLS B, DLS D);

CF 3: 0123456789120436597823491805678765904321948263715058302796147698541
032351709284640768132956951728403 (DLS C).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 5, a = 4, \rho = [1, 1, 2, 2, 2].$$

Method of finding:

Brute Force for horizontally symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

6.1. Loop-4 (1 different CF)

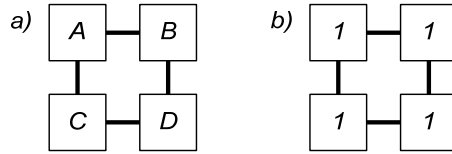


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 01234567891206798345367984251040852371965948360271786401395264521
79038951760482323905814678731925604 (DLS A, CF 1);

DLS 2: 01234567895389071426159263780496741852307450213968324876901527069
48153693182054780153946724867502391 (DLS B, CF 1);

DLS 3: 01234567898359071426159263780496741852307480213965324576901827069
48153693182054750183946724867502391 (DLS C, CF 1);

DLS 4: 01234567891206798345637984251040852671935948630271786401395234521
79068951730482626905814378731925604 (DLS D, CF 1).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120679834536798425104085237196594836027178640139526452179
038951760482323905814678731925604 (DLS A, DLS B, DLS C, DLS D).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 4, a = 4, \rho = [2, 2, 2, 2].$$

Method of finding:

search for partially symmetric DLSs («+»-symmetry, symmetric filled cells number $M = 70$ [7]) in combination with method of checking DLS for ODLS ODLS (Euler-Parker method, DLX).

Remark:

loop-4 structure also can be treated as rhombus-2.

6.2. Loop-4 (2 different CFs)

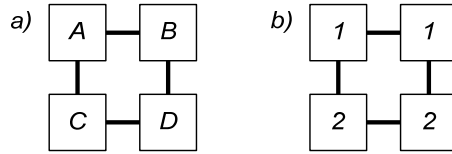


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 01234567893296017845783069512493472086512685930417197854320645627
81093840136957257198243606054172938 (DLS *A*, CF 1) – SODLS;

DLS 2: 01234567895471938620431672905819028745366094382175853016794292870
15364275964081376485032913865291407 (DLS *B*, CF 1) – SODLS;

DLS 3: 01234567895417938620437612905879028145366094382175853076194292810
75364275964081316485032973865297401 (DLS *C*, CF 2) – SODLS;

DLS 4: 01234567898296017345783069512493472086512685930417197854320645627
81093340186957257193248606054172938 (DLS *D*, CF 2) – SODLS.

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120657893460947325189745261803453980712659806134727358124
690867294035134170892652861395047 (DLS *A*, DLS *B*);

CF 2: 0123456789120467895395162430783687194205674893051240957826317931025
864846950132758723691402350817496 (DLS *C*, DLS *D*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 4, a = 4, \rho = [2, 2, 2, 2].$$

Method of finding:

Brute Force of SODLS.

6.3. Loop-4 (3 different CFs)

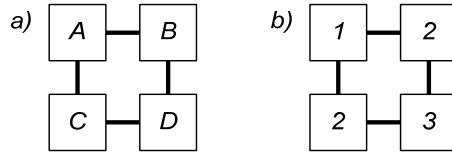


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 01234567891204635978203518469738479025167986543102647981025385607
29341461209783593582714605791368024 (DLS A, CF 1) – horizontal
symmetry;

DLS 2: 01234567898937264501758964013210647253983450189276429837106598765
32410534190862767028139542615097843 (DLS B, CF 2);

DLS 3: 01234567898945372601768953014210647253983270189456439826107598576
43210273190856454068179236512094837 (DLS C, CF 2);

DLS 4: 01234567891204635978203581469738479025167916543802647918025385607
29341468209713593512784605798361024 (DLS D, CF 3) – horizontal
symmetry.

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120463597820351846973847902516798654310264798102538560729
341461209783593582714605791368024 (DLS A);

CF 2: 0123456789123058796434791608525861972043960483521769520483718097613
425231679450875483216904785209136 (DLS B, DLS C);

CF 3: 0123456789120463597820315486975698271034486072931583520974617915364
802658790214394761832503749810526 (DLS D).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 4, a = 4, \rho = [2, 2, 2, 2].$$

Method of finding:

Brute Force for horizontally symmetric DLSs in combination with method of checking
DLS for ODLS (Euler-Parker method, DLX).

6.4. Loop-4 (4 different CFs)

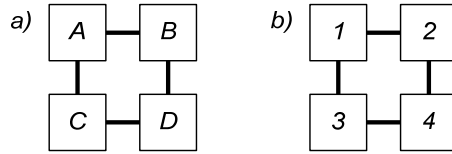


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 01234567891204379568789524301680567923414687531902693281547097610
84253357916082424189076355340628197 (DLS *A*, CF 1);

DLS 2: 01234567892476598301825017963439476150289361827450508976214365142
03897473208196578953402161608934572 (DLS *B*, CF 2);

DLS 3: 01234567892476598301825017963439476150289361827450508976214375142
03896463208197568953402171708934562 (DLS *C*, CF 3);

DLS 4: 01234567891204379568789524301680567923414617538902693218547097680
14253357986012424819076355340621897 (DLS *D*, CF 4).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120437956878952430168056792341468753190269328154709761084
253357916082424189076355340628197 (DLS *A*);

CF 2: 0123456789120453896726709851345489270613395682704165987134209015364
872784160239543670912588732149506 (DLS *B*);

CF 3: 0123456789120497863567308251945617392048437150982639426875109068714
352285604197384951302677589263401 (DLS *C*);

CF 4: 0123456789120437956878952430168056792341461753890269321854709768014
253357986012424819076355340621897 (DLS *D*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 4, a = 4, \rho = [2, 2, 2, 2].$$

Method of finding:

search for partially generalized symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

7. Treshka (1:3 structure)

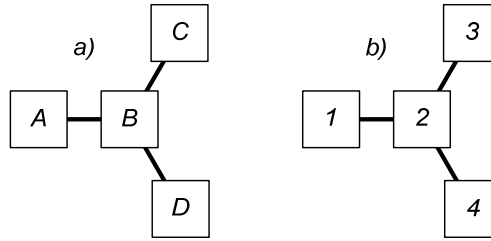


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 01234567891237098546409637182596845130725968702431345928061787016
35294237596410875128493606840127953 (DLS B, CF 1);

DLS 2: 01234567896951780432924016857335769028418639514207481762395013648
97025578204139620983756147405239168 (DLS A, CF 2);

DLS 3: 01234567899304872165125893704684970653212075648913453671920879621
84530671920385458403216973681590472 (DLS C, CF 3);

DLS 4: 01234567899374802165125893704684970653212705648913453671920879621
84530601927385458403216973681590472 (DLS D, CF 4).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789123709854640963718259684513072596870243134592806178701635
294237596410875128493606840127953 (DLS A);

CF 2: 0123456789120453896760852973144516982073349271560876398401522960173
845984762153087513042965378069421 (DLS B);

CF 3: 0123456789120468793593625081743791064258854972130654378906126980135
427405627389126783195407815942063 (DLS C);

CF 4: 0123456789120468793595623081743791064258834972150654378906126980135
427405627389126785193407815942063 (DLS D).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 4, a = 3, \rho = [1, 1, 1, 3].$$

Method of finding:

horizontally symmetric DLSs canonization, search for partially generalized symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

8.1. Symmetric four (1:4 structure)

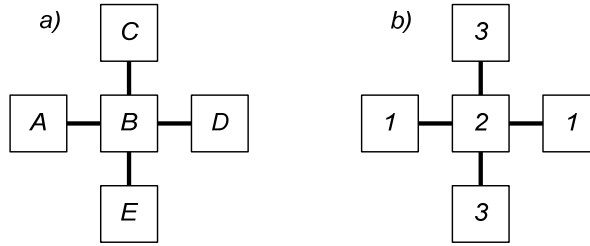


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891204365978235109846785967230413749180526406827139574806
39152691254780396758142305837902614 (DLS *B*, CF 1) – horizontal
symmetry;
- DLS 2: 01234567894850279613603718259496125478308265903471170436592839768
14205538972014674916380522548091367 (DLS *A*, CF 2);
- DLS 3: 01234567894859270613693718250496125478308265903471179436502830768
14295538072914674016389522548091367 (DLS *C*, CF 3);
- DLS 4: 01234567896830279415504718269396125478308256904371170436592849758
13206358972016474916380522368091547 (DLS *D*, CF 2);
- DLS 5: 01234567896839270415594718260396125478308256904371179436502840758
13296358072916474016389522368091547 (DLS *E*, CF 3).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789120436597823510984678596723041374918052640682713957480639
152691254780396758142305837902614 (DLS *B*);
- CF 2: 0123456789120467983530981645272836791054795132840693158472604780935
612867950214354620839716547210398 (DLS *A*, DLS *D*);
- CF 3: 0123456789120457896390316278544765982031389671524079183604252380194
576657984310284520316975647209318 (DLS *C*, DLS *E*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 5, a = 4, \rho = [1, 1, 1, 1, 4].$$

Method of finding:

search for horizontally symmetric and partially generalized symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

Remark:

first time horizontally symmetric string-inverse DLS corresponding for 1:4 structure was described in [4], but this article has only 3 ODLS for them; 4-th ODLS was found and described in article [8].

8.2. Asymmetric four (1:4 structure)

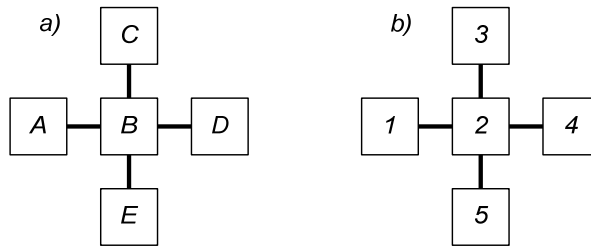


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891204379568348509217657392648019561708243784651039260529
83417297084163543186279508697135024 (DLS *A*, CF 1);
- DLS 2: 01234567899546287031603754981243587126903602974158278516390478906
21345826109547314798305265914308267 (DLS *B*, CF 2);
- DLS 3: 01234567899546187032603754982143587216903602974158178526390478906
12345826109547324798305165914308267 (DLS *C*, CF 3);
- DLS 4: 01234567894596287031603754981293587126403602974158278516390478406
21395826109547314798305265914308267 (DLS *D*, CF 4);
- DLS 5: 01234567894596187032603754982193587216403602974158178526390478406
12395826109547324798305165914308267 (DLS *E*, CF 5).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789120437956834850921765739264801956170824378465103926052983
417297084163543186279508697135024 (DLS *A*);
- CF 2: 0123456789120453796846872935019815672430609231584785710642935438709
612374698012523591480767960821354 (DLS *B*);
- CF 3: 0123456789120453896727586903419687214530643517980239460872154892703
156501984267375603214988371965024 (DLS *C*);
- CF 4: 0123456789120458796346872935019815672430609231584735710642985438709
612874693012523591480767960821354 (DLS *D*);
- CF 5: 0123456789120453896727586903419867214530643517980239460872154692703
158501984267375803214968371965024 (DLS *E*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 5, a = 4, \rho = [1, 1, 1, 1, 4].$$

Method of finding:

search for partially generalized symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

9. Five (1:5 structure)

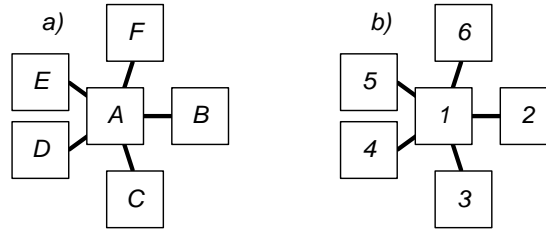


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891204879563931768042537950482162586937104794826135086315
04972647032589140597126385862193047 (DLS A, CF 1);
- DLS 2: 01234567898596034271673259184054897120637864105932925738061419456
27308201896345736708491254301278596 (DLS B, CF 2);
- DLS 3: 01234567893596084271678259134054397120687864105932925783061419456
27803201896345786703491254301278596 (DLS C, CF 3);
- DLS 4: 01234567893596084271678259314054197320687864305912925781063419456
27803203896145786701493254301278596 (DLS D, CF 4);
- DLS 5: 01234567893796084251658279134074593120685864103972927583061419476
25803201896743586305491274301278596 (DLS E, CF 5);
- DLS 6: 01234567893796084251658279314074591320685864301972927581063419476
25803203896741586105493274301278596 (DLS F, CF 6).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789120487956393176804253795048216258693710479482613508631504
972647032589140597126385862193047 (DLS A);
- CF 2: 0123456789123067594896821043578574032196749538162048165930723961847
205230796851457492108636058729431 (DLS B);
- CF 3: 0123456789123067594896871043528574032196249538167048165930273961847
205730296851457492108636058729431 (DLS C);
- CF 4: 0123456789120453796839708651242489670513906732485156487132906395108
472853124960778520913464716982035 (DLS D);
- CF 5: 0123456789120657893440372658916481927350935871042638790426155614809
273796018354285423910672795634108 (DLS E);
- CF 6: 0123456789120458796330627458919647801325538917264065183902744870923
516795106843284362190572795634108 (DLS F).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 6, a = 5, \rho = [1, 1, 1, 1, 1, 5].$$

Method of finding:

search for partially central symmetric DLSs, symmetric filled cells number $M = 60$, in combination with method of checking DLS for ODLs (Euler-Parker method, DLX).

10.1. Asymmetric six (1:6 structure)

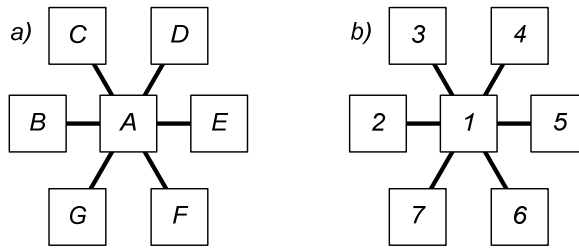


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567896582097413289163504773085621948917340256405987136232769
14508143528967097641038255640728931 (DLS A, CF 1);
- DLS 2: 01234567891206783945534890726146398215077562138094291756043884906
75312907431285638512496706785094123 (DLS B, CF 2);
- DLS 3: 01234567891806723945534290786146392815077568132094291756043884906
75312907431825632518496706785094123 (DLS C, CF 3);
- DLS 4: 01234567891806273945534790286146397815022568137094791256043884906
25317907431825632518496706785094123 (DLS D, CF 4);
- DLS 5: 01234567891206789345594830726146918235077562918034231756049884306
75912907413285638592416706785094123 (DLS E, CF 5);
- DLS 6: 01234567891806729345594230786146912835077568912034231756049884306
75912907413825632598416706785094123 (DLS F, CF 6);
- DLS 7: 01234567891806279345594730286146917835022568917034731256049884306
25917907413825632598416706785094123 (DLS G, CF 7).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789123406897526953840176487531290795084213640762198533718695
402956172034883429075615809173624 (DLS A);
- CF 2: 0123456789120678394553489072614639821507756213809429175604388490675
312907431285638512496706785094123 (DLS B);
- CF 3: 0123456789123450986779518624034368721095307968415224903156786842937
510850627394197851402365617098324 (DLS C);
- CF 4: 0123456789123450986779418625035368721094307968415224903156786852937
410850627394197851402364617098325 (DLS D);
- CF 5: 0123456789120678934559483072614691823507756291803423175604988430675
912907413285638592416706785094123 (DLS E);
- CF 6: 0123456789123470986570523614984796528031397081564286192745039547683
120240813795663859402175861092374 (DLS F);

CF 7: 0123456789123078495647958026135418397062238461950796710482357062135
498854796132068095231743956270841 (DLS G).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 7, a = 6, \rho = [1, 1, 1, 1, 1, 6].$$

Method of finding:

search for partially central symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

10.2. Symmetric six (1:6 structure)

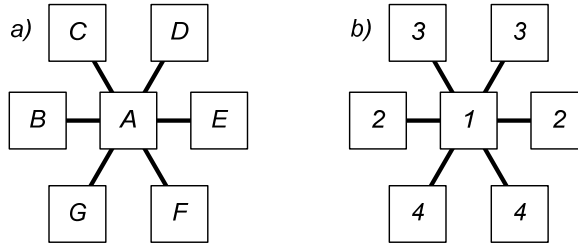


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891234095678234081956798765432108765904321649127805335087
21946501763289476591804324982367105 (DLS *A*, CF 1) – horizontal
symmetry, string-inverse square;
- DLS 2: 01234567898465172930609723815427518640931649320875321458960753860
97412490271356898706453217538901246 (DLS *B*, CF 2);
- DLS 3: 01234567896512309847829504731623047156981970628435563718492098465
73102478196025330682915747459832061 (DLS *C*, CF 3);
- DLS 4: 01234567892510967843386259407110348259674651739208970518263479862
43510647930812552480713968397610452 (DLS *D*, CF 3);
- DLS 5: 01234567899607284351548167209360953184274219760538293014587678520
93164134682790587645392103578901642 (DLS *E*, CF 2);
- DLS 6: 01234567897901384526857219063423549081675087612943961853740242608
75391184906327567352498103496721058 (DLS *F*, CF 4);
- DLS 7: 01234567893745168902563908724123819054676507832194795264183080642
19375427639051898105746231498723056 (DLS *G*, CF 4).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789123409567823408195679876543210876590432164912780533508721
946501763289476591804324982367105 (DLS *A*);
- CF 2: 0123456789120453896743718092566948075321865294701375391624089810623
574548679013227953816403067214895 (DLS *B*, DLS *E*);
- CF 3: 0123456789123408957648519203675076831492249710365839806752149612748
035634851792075093628418765294103 (DLS *C*, DLS *D*);
- CF 4: 0123456789120453896724968051733865172094751238964059487613029630247
851875192043663870942154079613528 (DLS *F*, DLS *G*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 7, a = 6, \rho = [1, 1, 1, 1, 1, 1, 6].$$

Method of finding:

Brute Force for horizontally symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

11. Seven (1:7 structure)

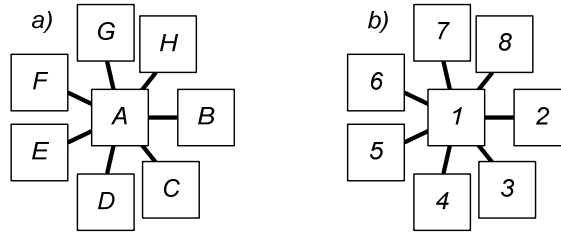


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891204379568394071582690318476525897620143756809243164751
38290261958430783529610744786203915 (DLS A, CF 1);
- DLS 2: 01234567893967541802865907231424187059364572183690908623714572319
64058534069827117948205636805319427 (DLS B, CF 2);
- DLS 3: 01234567893961542807805926731474186259301540783296968213407527349
01658537609842142978105636805379142 (DLS C, CF 3);
- DLS 4: 01234567893961542807805296731474186952301540783926268913407597342
01658537602849142978105636805379142 (DLS D, CF 4);
- DLS 5: 01234567893961542807805296731474186952301540783962628913407597342
01658537602849146978105232805379146 (DLS E, CF 5);
- DLS 6: 01234567893961542807805926731474186259301540783296968213407527340
91658537690842142078195636895370142 (DLS F, CF 6);
- DLS 7: 01234567893961542807865902731474182059361542783690908613427527349
61058537069842142978105636805379142 (DLS G, CF 7);
- DLS 8: 01234567893961542807865209731474189052361549783620208613497597342
61058537062849142978105636805379142 (DLS H, CF 8).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789120437956839407158269031847652589762014375680924316475138
290261958430783529610744786203915 (DLS A);
- CF 2: 0123456789123058964738927650144586271930741832059697046132585967804
321265914780363450981728071932465 (DLS B);
- CF 3: 0123456789120459786394603281752839670514398674205187452196304698135
207657180439273520819465017963428 (DLS C);
- CF 4: 0123456789123460985735879124062906341578841973526048502976139378564
021576208319460451789327691820345 (DLS D);
- CF 5: 0123456789123708964547506913282346578091796482513090827134568591364
207680523791436189405725479102863 (DLS E);

CF 6: 0123456789120479836535978610429086347521795812463048305792165461203
978674201589323196804578675932104 (DLS *F*);
CF 7: 0123456789120458796394603251782839670514358674209157482196304695138
207697180435273520918468017963425 (DLS *G*);
CF 8: 0123456789120654897347380921655917364802865413902794726805313890275
416738592164065417032982069817354 (DLS *H*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 8, a = 7, \rho = [1, 1, 1, 1, 1, 1, 7].$$

Method of finding:

search for partially horizontally symmetric DLSs, symmetric filled cells number $M = 60$,
in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

12.1. Symmetric eight (1:8 structure)

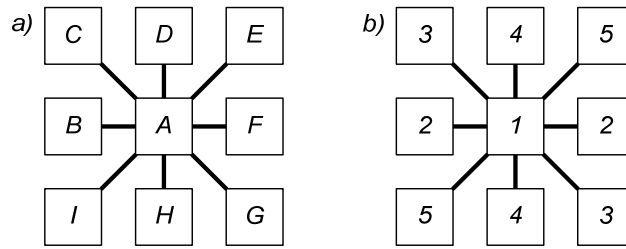


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891230549678496827130568749052137359180462948763215035917
28046874609352150123678942605814937 (DLS *A*, CF 1) – horizontal
symmetry;
- DLS 2: 01234567897901368542347618925085326974011760534928289504361762589
01374401927586396478120355384720196 (DLS *B*, CF 2);
- DLS 3: 01234567892901368547347618925085326974011760534928789504361262589
01374401972586396478120355384270196 (DLS *C*, CF 3);
- DLS 4: 01234567897901368542347681925015326974088760534921289504361762589
01374401927586396471820355384720196 (DLS *D*, CF 4);
- DLS 5: 01234567892901368547347681925015326974088760534921789504361262589
01374401972586396471820355384270196 (DLS *E*, CF 5);
- DLS 6: 01234567897541368902947018325689520376411705649328283659401752689
01473631427089546978125303089725164 (DLS *F*, CF 2);
- DLS 7: 01234567892541368907947018325689520376411705649328783659401252689
01473631472089546978125303089275164 (DLS *G*, CF 3);
- DLS 8: 01234567897541368902947081325619520376488705649321283659401752689
01473631427089546971825303089725164 (DLS *H*, CF 4);
- DLS 9: 01234567892541368907947081325619520376488705649321783659401252689
01473631472089546971825303089275164 (DLS *I*, CF 5).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789123054967849682713056874905213735918046294876321503591728
046874609352150123678942605814937 (DLS *A*);
- CF 2: 0123456789120436795889602753419837520614765918243024986315073582904
176601574982357468130924371098265 (DLS *B*, DLS *F*);
- CF 3: 0123456789120463795889302756419867520314735918246024983615073582904
176601574982357468130924671098235 (DLS *C*, DLS *G*);

CF 4: 0123456789123409587664879012537605834921951278364047981620353850219
467896154730223496705185076328194 (DLS *D*, DLS *H*);
CF 5: 0123456789123409587665879012437605834921941278365047981620353840219
567896154730223596704185076328194 (DLS *E*, DLS *I*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 9, a = 8, \rho = [1, 1, 1, 1, 1, 1, 1, 8].$$

Method of finding:

Brute Force for horizontally symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

12.2. Asymmetric eight (1:8 structure)

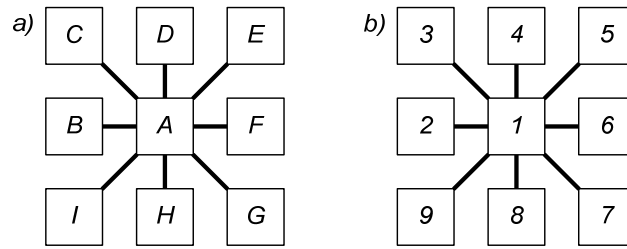


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891204573968573219480630189456728946710253485136209775608
29134937560842164892375102697081345 (DLS A, CF 1);
- DLS 2: 01234567892385697041106483957295471803266418572930397021546842913
08657783906421556027418938756923104 (DLS B, CF 2);
- DLS 3: 01234567892385967041106483957295471803266418572930397021546842913
08657783609421556027418938759623104 (DLS C, CF 3);
- DLS 4: 01234567892385697014406183957295471803266418572930397024516812943
08657783906124556027148938756923401 (DLS D, CF 4);
- DLS 5: 01234567892385967014406183957295471803266418572930397024516812943
08657783609124556027148938759623401 (DLS E, CF 5);
- DLS 6: 01234567892385697041186403957295471803266410572938397821546042913
08657703986421556027418938756923104 (DLS F, CF 6);
- DLS 7: 01234567892385967041186403957295471803266410572938397821546042913
08657703689421556027418938759623104 (DLS G, CF 7);
- DLS 8: 01234567892385697014486103957295471803266410572938397824516012943
08657703986124556027148938756923401 (DLS H, CF 8);
- DLS 9: 01234567892385967014486103957295471803266410572938397824516012943
08657703689124556027148938759623401 (DLS I, CF 9).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789120457396857321948063018945672894671025348513620977560829
134937560842164892375102697081345 (DLS A);
- CF 2: 0123456789120463597878395406124391728065256890314769471825035612097
834875036942194862713503075814296 (DLS B);
- CF 3: 0123456789120463597878395406124591728063238690154769471823055612097
834875036942194682731503075814296 (DLS C);

CF 4: 0123456789120463597839528674014319270865804572319674601893525678901
 234983754261025963180476781094523 (DLS *D*);
CF 5: 0123456789120463597839528674014319270865806572319476401893525478901
 236983754261025963180476781094523 (DLS *E*);
CF 6: 0123456789120463597828395406174391278065756890314269471825035612097
 834875036942194867213503075814296 (DLS *F*);
CF 7: 0123456789120463597828395406174591278063738690154269471823055612097
 834875036942194687231503075814296 (DLS *G*);
CF 8: 0123456789120436895798621753408347091562641952087370586342912670849
 135398571240657962830144531907628 (DLS *H*);
CF 9: 0123456789120463597823469085176958721304549718206335192708467860349
 152873509462196825174304071863295 (DLS *I*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 9, a = 8, \rho = [1, 1, 1, 1, 1, 1, 1, 8].$$

Method of finding:

search for partially central symmetric DLSs, symmetric filled cells number $M = 60$, in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

13. Symmetric ten (1:10 structure)

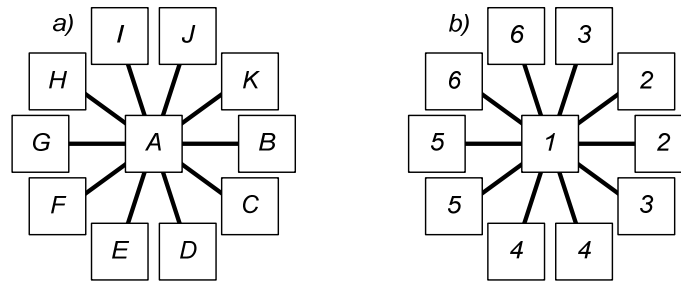


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891204879563754291830627810356948639507421945826103750976
43812631578294038601942754976320158 (DLS *A*, CF 1) – generalized
symmetry (4,31,31);
- DLS 2: 01234567893851694207549087263190162485734985713062163752049867023
81954854910732623789651407264039815 (DLS *B*, CF 2);
- DLS 3: 01234567894851693207539087264190162485733985714062164752039867023
81954853910742624789651307264039815 (DLS *C*, CF 3);
- DLS 4: 01234567893851694207549017263890162485734985713062863752049167023
81954154980732623789651407264039815 (DLS *D*, CF 4);
- DLS 5: 01234567894851693207539017264890162485733985714062864752039167023
81954153980742624789651307264039815 (DLS *E*, CF 4);
- DLS 6: 01234567893851694207549708263190162485734905713862163052749867823
01954854917032623789651407264839015 (DLS *F*, CF 5);
- DLS 7: 01234567894851693207539708264190162485733905714862164052739867823
01954853917042624789651307264839015 (DLS *G*, CF 5);
- DLS 8: 01234567893851694207549780263190162485734985713062163052749867023
81954854917032623789651407264039815 (DLS *H*, CF 6);
- DLS 9: 01234567894851693207539780264190162485733985714062164052739867023
81954853917042624789651307264039815 (DLS *I*, CF 6);
- DLS 10: 0123456789385169420754971026389016248573498571306286305274916702
381954154987032623789651407264039815 (DLS *J*, CF 3);
- DLS 11: 0123456789485169320753971026489016248573398571406286405273916702
381954153987042624789651307264039815 (DLS *K*, CF 2).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120487956375429183062781035694863950742194582610375097643
812631578294038601942754976320158 (DLS *A*);

CF 2: 0123456789120436895768127093453498172506265091743853798401624736285
091894752361090856312747561094823 (DLS *B*, DLS *K*);

CF 3: 0123456789120437956836491802574058937621638570419229175638049871245
036573681294074906283158562091473 (DLS *C*, DLS *J*);

CF 4: 0123456789120458936763572981047961835240459867203197823406155470163
892864570192338190245762036917458 (DLS *D*, DLS *E*);

CF 5: 0123456789120476893575398401625698274310681592307439670125489046537
821235068149747821096538471395206 (DLS *F*, DLS *G*);

CF 6: 0123456789120458936736570981247961835240459867203197823406155470163
892834572190668192045732036917458 (DLS *H*, DLS *I*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 11, a = 10, \rho = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 10].$$

Method of finding:

search for generalized symmetric DLSs with (4,31,31) generalized symmetry in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

14. Rhombus-3

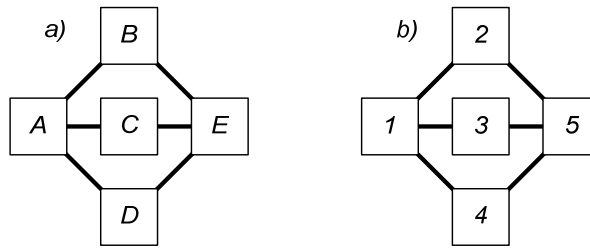


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891237048965605931247889046215377861935204367528409124908
67153458617932097485036125312790846 (DLS *A*, CF 1);
- DLS 2: 01234567894360295871187296035435418729062937684510509812764396157
48032820951346774560312986784309125 (DLS *B*, CF 2);
- DLS 3: 01234567894860295371137296085425418739068937624510509813764296157
42038320951846774560812936784309125 (DLS *C*, CF 3);
- DLS 4: 01234567894860295371137296085425418739068937642510509813764296157
24038340951826772560814936784309125 (DLS *D*, CF 4);
- DLS 5: 01234567891237048965605931247889045216377861935204357628409124908
67153468517932097486035125312790846 (DLS *E*, CF 5).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789123704896560593124788904621537786193520436752840912490867
153458617932097485036125312790846 (DLS *A*);
- CF 2: 0123456789120467895369875304124831027695275936180456982430713570819
246741698253093427051688065194327 (DLS *B*);
- CF 3: 0123456789120458793629370156486895730412941862305753498721608761294
503367014982575863012944052968371 (DLS *C*);
- CF 4: 0123456789123406985759468170237308291546375968241080123759644897523
601956074813264751302982681904375 (DLS *D*);
- CF 5: 0123456789123679084567489321504815203697307182596493645712082587649
031540916837276920845138950317426 (DLS *E*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 5, a = 6, \rho = [2, 2, 2, 3, 3].$$

Method of finding:

search for partially central symmetric DLSs, symmetric filled cells number $M = 60$, in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

15.1. Rhombus-4 (6 CF)

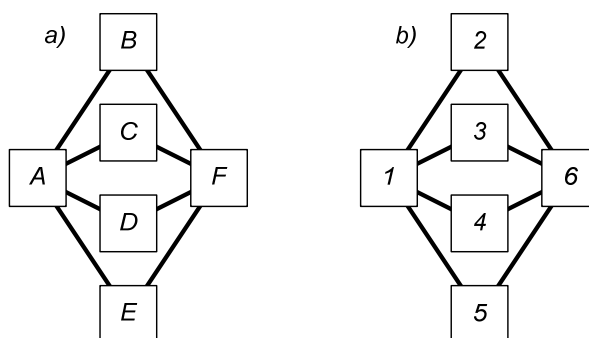


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891204368957941827536023579804168960527134584679120360718
43592378910462576950328414532619078 (DLS A, CF 1);
- DLS 2: 01234567893769081245608594213746928753019548103672741063952859072
64813283451796012763980548351720496 (DLS B, CF 2);
- DLS 3: 01234567893796081245608594213749628753019548103672741063952856072
94813283451796012793680548351720496 (DLS C, CF 3);
- DLS 4: 01234567893769081245608594713246928753019548103627241063957859072
64813783451296012763980548351720496 (DLS D, CF 4);
- DLS 5: 01234567893796081245608594713249628753019548103627241063957856072
94813783451296012793680548351720496 (DLS E, CF 5);
- DLS 6: 01234567891204568937941827356023579804168960327154384679120560718
45392578910462376950328414532619078 (DLS F, CF 6).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789120436895794182753602357980416896052713458467912036071843
592378910462576950328414532619078 (DLS A);
- CF 2: 0123456789120487953660485371929736245801359178426073690184258452963
017587062194326871903544915302678 (DLS B);
- CF 3: 0123456789120679834550648731927451980623964753280187352490164398167
250387201596425896014376910324578 (DLS C);
- CF 4: 0123456789123459067847629083518347621095907184356259862174307698135
204280536491735190728466450789123 (DLS D);
- CF 5: 0123456789123057964830596148728974362501459710832624657801939786243
015681203795473418952605608921437 (DLS E);
- CF 6: 0123456789120456893794182735602357980416896032715438467912056071845
392578910462376950328414532619078 (DLS F).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 6, a = 8, \rho = [2, 2, 2, 2, 4, 4].$$

Method of finding:

search for partially central symmetric DLSs, symmetric filled cells number $M = 60$, in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

15.2. Rhombus-4 (5 CF)

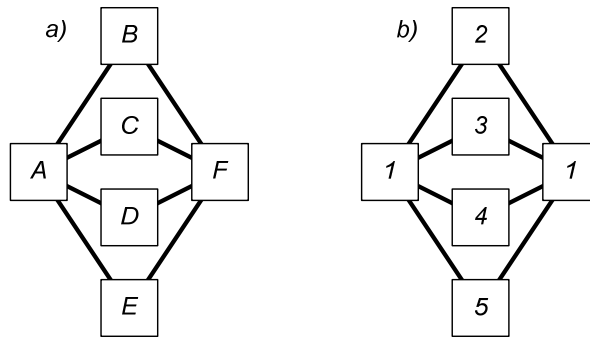


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891234095678341790285659768134029758361240456018932768052
47913864972053170826341952391578064 (DLS A, CF 1);
- DLS 2: 01234567893740819526826409537165192708437305184962983754261020516
38497147690325856987210344982367105 (DLS B, CF 2) – horizontal
symmetry;
- DLS 3: 01234567893740819526826409537165197208432305184967983254761070516
38492147690325856982710344987362105 (DLS C, CF 3) – horizontal
symmetry;
- DLS 4: 01234567893750819426826409537164192708537305184962983754261020416
38597157690324856987210344982367105 (DLS D, CF 4) – horizontal
symmetry;
- DLS 5: 01234567893750819426826409537164197208532305184967983254761070416
38592157690324856982710344987362105 (DLS E, CF 5) – horizontal
symmetry;
- DLS 6: 01234567891234095678341790285679568132049578361420276018934568025
74913864972053140856371925391248067 (DLS F, CF 1).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789123409567834179028565976813402975836124045601893276805247
913864972053170826341952391578064 (DLS A, DLS F);
- CF 2: 0123456789120478963595703421682915634870569280734170382659148347091
256375691840248615230976489170523 (DLS B);
- CF 3: 0123456789120436597837850941265379810264964172853028605493177958631
402403218769584962730516517902843 (DLS C);
- CF 4: 0123456789120478963595703481628915634270569280734170382659142347091
856375691240848615230976489170523 (DLS D);
- CF 5: 0123456789120463597837850941265679810234934172856028605493177958361
402403218769584962730516517902843 (DLS E).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 6, a = 8, \rho = [2, 2, 2, 2, 4, 4].$$

Method of finding:

Brute Force for horizontally symmetric DLSSs in combination with method of checking DLS for ODLs (Euler-Parker method, DLX).

15.3. Rhombus-4 (4 CF)

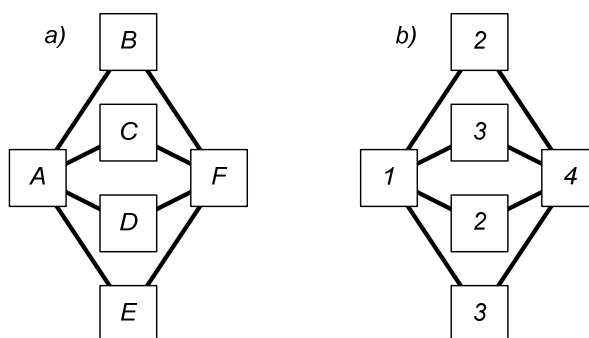


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891204635978371054982625398106475896723014408736219576419
08532935827146084620973516975184203 (DLS A, CF 1) – horizontal
symmetry;
- DLS 2: 01234567899845367120857963041279561238044381902567126074935867982
15043301758429626048719355432098671 (DLS B, CF 2);
- DLS 3: 01234567899845367120857963041270561238944381902567126074935867082
15943391758420626948710355432098671 (DLS C, CF 3);
- DLS 4: 01234567899782364510785963024159167834022347908165146052937865948
71023307514289646082159378231097654 (DLS D, CF 2);
- DLS 5: 01234567899782364510785963024150167834922347908165146052937865048
71923397514280646982150378231097654 (DLS E, CF 3);
- DLS 6: 01234567891294635078371054982625398106475806723914498736210576410
98532935827146084629073516075184293 (DLS F, CF 4) – horizontal
symmetry.

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789120463597837105498262539810647589672301440873621957641908
532935827146084620973516975184203 (DLS A);
- CF 2: 0123456789120457396854807391268549260371706834159297368124503895627
014497218560363510982472617904835 (DLS B, DLS D);
- CF 3: 0123456789120436597823159048673469780152789154203656801793249742638
510493782160585760932416058217493 (DLS C, DLS E);
- CF 4: 0123456789120463597829315486074698271035948672315065798102433750189
426804236759173150948625867902314 (DLS F).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 6, a = 8, \rho = [2, 2, 2, 2, 4, 4].$$

Method of finding:

Brute Force for horizontally symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

16.1. Symmetric fish

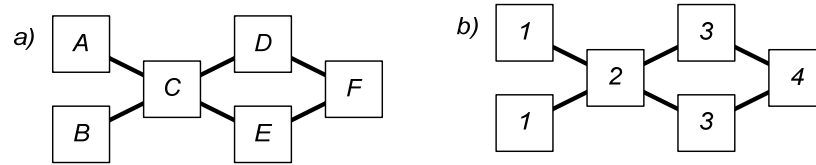


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891204367958879652403169451823704510739826205781369493716
08542386927041554380912677682945103 (DLS *A*, CF 1);
- DLS 2: 01234567894731908625241963085778943650128056273491697518420352608
19374134209756896875421303508721946 (DLS *C*, CF 2) – horizontal
symmetry;
- DLS 3: 01234567891204367958879652403196451823704510739826205781369463719
08542386927041554380912677982645103 (DLS *D*, CF 3);
- DLS 4: 01234567891402365978869574302192671845033710629845503681249775419
38260485927031623780916546984507132 (DLS *B*, CF 1);
- DLS 5: 01234567891402365978869574302192671845303710629845503681249775419
08263485927031623780916546984537102 (DLS *E*, CF 3);
- DLS 6: 01234567894731908625741963085228943650178056273491697518420352608
19374134709256896825471303508721946 (DLS *F*, CF 4) – horizontal
symmetry.

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789120436795887965240316945182370451073982620578136949371608
542386927041554380912677682945103 (DLS *A*, DLS *B*);
- CF 2: 0123456789123054967846789012355961278304875936042175961830422805634
917934781256060147258933482097156 (DLS *C*);
- CF 3: 0123456789120436795887965240319645182370451073982620578136946371908
542386927041554380912677982645103 (DLS *D*, DLS *E*);
- CF 4: 0123456789120487963589367145022579108346908562147376580432914897365
120571093286434625970186341280957 (DLS *F*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 6, a = 6, \rho = [1, 1, 2, 2, 2, 4].$$

Method of finding:

Brute Force for horizontally symmetric DLSs in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

16.2. Asymmetric fish

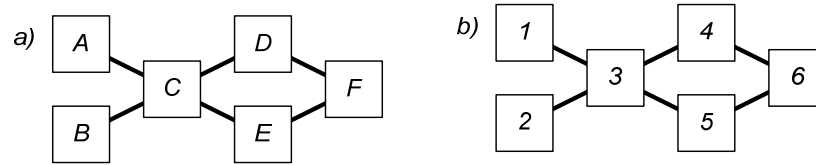


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891204367958491257803656791438023758614290739128064520378
95164958603142784659023716840729513 (DLS A, CF 1);
- DLS 2: 01234567899457028613526190437840183925678376541902254087913667952
13840380916725416327804957984635021 (DLS C, CF 2);
- DLS 3: 01234567891205367948491257803656791438023748615290739128065420378
94165948603152785649023716850729413 (DLS D, CF 3);
- DLS 4: 01234567891205367948498257103656791438023741685290739821065420378
94165941603852785649023716850729413 (DLS E, CF 4);
- DLS 5: 01234567891204367958498257103656791438023751684290739821064520378
95164951603842784659023716840729513 (DLS B, CF 5);
- DLS 6: 01234567899457028613526910437840183925678376549102254087193667952
13840380196725416327804957984635021 (DLS F, CF 6).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789120458396763890475128975612340469713025827368954019860374
125501872963434529018767541268093 (DLS A);
- CF 2: 0123456789120437985658917406232476108395963581740240872359613950681
247674209351883695241707518962034 (DLS B);
- CF 3: 0123456789120436795849125780365679143802375861429073912806452037895
164958603142784659023716840729513 (DLS C);
- CF 4: 0123456789120463795837819602452059783164957210843659368410274697315
802731852469084650923716840279513 (DLS D);
- CF 5: 0123456789120436795849825710365679143802375168429073982106452037895
164951603842784659023716840729513 (DLS E);
- CF 6: 0123456789120458396763890475128975612340269713045847368952019860374
125501872963434529018767541268093 (DLS F).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 6, a = 6, \rho = [1, 1, 2, 2, 2, 4].$$

Method of finding:

search for partially generalized symmetric DLSs with (4,31,31) symmetry, symmetric filled cells number $M = 70$, in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

17. Cross

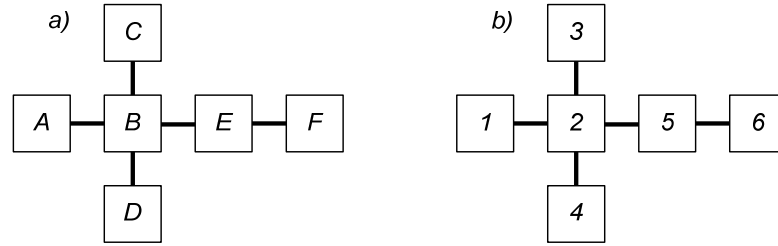


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891204678953754201369837682915044079385261938576041268105
29347593684712084519320762697104835 (DLS *A*, CF 1);
- DLS 2: 01234567894638597012527916083410958724636842903175871634529029847
31506946021835735076849217351029648 (DLS *B*, CF 2);
- DLS 3: 01234567898204671953754208369137612985044079315268938576041268105
29347593614782014589320762697804135 (DLS *C*, CF 3);
- DLS 4: 01234567898207641953754208369137612985044079315268938576041268105
29347593617482014589320762694807135 (DLS *E*, CF 4);
- DLS 5: 01234567891207648953754201369837682915044079385261938576041268105
29347593687412084519320762694107835 (DLS *D*, CF 5);
- DLS 6: 01234567894618597032527936081410958724636842901375873614529029847
13506946023815735076849217351029648 (DLS *F*, CF 6).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789120467895375420136983768291504407938526193857604126810529
347593684712084519320762697104835 (DLS *A*);
- CF 2: 0123456789123057986479623041582749680531630184592745172380968095761
342567809241398541236703486917205 (DLS *B*);
- CF 3: 0123456789120657983437149856208965037142639784250145821903679648703
215785026149350316249782479318056 (DLS *C*);
- CF 4: 0123456789120479865346598273106548371092901268354757962348013870169
425296754013884319052767385012964 (DLS *E*);
- CF 5: 0123456789120459867396582104373417902865256983701487326495016841073
952798536412040761253985390781246 (DLS *D*);
- CF 6: 0123456789123408957635816724902679540318409872563159071648237415893
062986230715463502189478746931205 (DLS *E*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 6, a = 5, \rho = [1, 1, 1, 1, 2, 4].$$

Method of finding:

search for partially central symmetric DLSSs, symmetric filled cells number $M = 60$, in combination with method of checking DLS for ODLs (Euler-Parker method, DLX).

18. Flyer

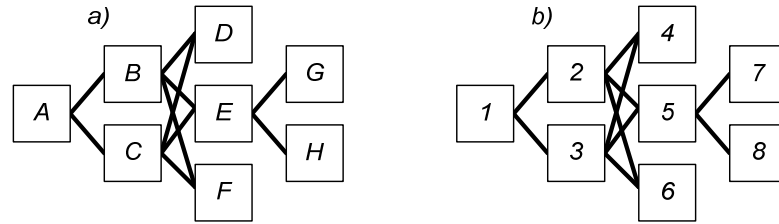


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891204638957409128537669475018237359840162583671249085709
63214978532460136120975482468179035 (DLS A, CF 1);
- DLS 2: 01234567894387092615165987340292346185706578904231201536984734921
87056786024519387065319245941720368 (DLS B, CF 2);
- DLS 3: 01234567894837092615165937840292846135706578904231201586934734921
87056736024519887065319245941720863 (DLS C, CF 3);
- DLS 4: 01234567891204638957409128537669175048237359810462583674219085709
63241978532160436420975182468179035 (DLS D, CF 4);
- DLS 5: 01234567891204638957409128537669475218037359842160583671049285709
63214978530462136120975482468179035 (DLS E, CF 5);
- DLS 6: 01234567891204638957409128537669175248037359812460583674019285709
63241978530162436420975182468179035 (DLS F, CF 6);
- DLS 7: 01234567894387092615165087349292346185706578904231201536984734021
87956786924510387965310245941720368 (DLS G, CF 7);
- DLS 8: 01234567894837092615165037849292846135706578904231201586934734021
87956736924510887965310245941720863 (DLS H, CF 8).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789120463895740912853766947501823735984016258367124908570963
214978532460136120975482468179035 (DLS A);
- CF 2: 0123456789120458397626579148039748631520806534719243912780655470169
238793280564135860924176819720354 (DLS B);
- CF 3: 0123456789120458397626579418039718634520806531749243912780655470169
238793280561435860921476849720351 (DLS C);
- CF 4: 0123456789120463895740912853766917504823735981046258367421908570963
241978532160436420975182468179035 (DLS D);
- CF 5: 0123456789120463895740912853766947521803735984216058367104928570963
214978530462136120975482468179035 (DLS E);

CF 6: 0123456789120463895740912853766917524803735981246058367401928570963
241978530162436420975182468179035 (DLS *F*);
CF 7: 0123456789120657943834672158907695834021954812760348326901752970348
516801976235467510839425384901267 (DLS *G*);
CF 8: 0123456789120657943834672158907685934021854912760348326901752970348
516901876235467510839425394801267 (DLS *H*).

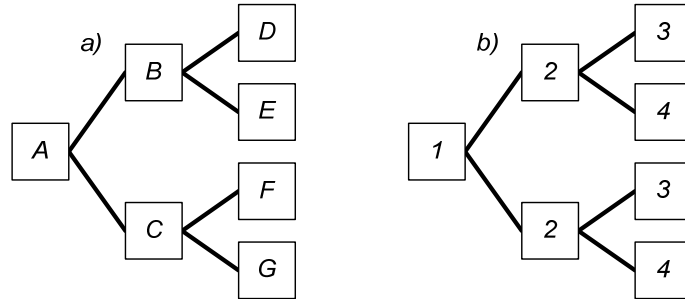
Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 8, a = 10, \rho = [1, 1, 2, 2, 2, 4, 4, 4].$$

Method of finding:

search for partially central symmetric DLSs, symmetric filled cells number $M = 60$, in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

20. Tree-1



DLSs within combinatorial structure:

- DLS 1: 01234567891204679853847591063270612385949836521407379208416565478
93210261034597853897620414958107326 (DLS A, CF 1) – generalized
symmetry (4,31,31);
- DLS 2: 01234567896375892041104962537829081476534582703916583726149072613
89504945607813236905148278714930265 (DLS B, CF 2);
- DLS 3: 01234567896475892031103962547829081476533582704916584726139072613
89504935607814246905138278714930265 (DLS C, CF 2);
- DLS 4: 01234567891204679853847591263070612385949836521407379208416565478
93012261034597853897602414958107326 (DLS D, CF 3);
- DLS 5: 01234567891234679850847591260370612385949806521437379208416565478
93012261034597853897602414958107326 (DLS E, CF 4);
- DLS 6: 01234567891204679853547891063270612385949836521407379208416565478
93210261034597883597620414985107326 (DLS F, CF 3);
- DLS 7: 01234567891208679453587491063270612385949436521807379208416565478
93210261034597883597620414985107326 (DLS G, CF 4).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789120467985384759106327061238594983652140737920841656547893
210261034597853897620414958107326 (DLS A);
- CF 2: 0123456789123406985774581906325796843210381297540646072813959381527
064654930217820756389418960714523 (DLS B, DLS C);
- CF 3: 0123456789120458793656489302176459108372936074152887920136454876325
190391567280425378940617081269453 (DLS D, DLS F);
- CF 4: 0123456789120458793656489302176459108372236074159887920136454876395
120391567280495378240617081269453 (DLS E, DLS G).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 7, a = 6, \rho = [1, 1, 1, 1, 2, 3, 3].$$

Method of finding:

search for generalized symmetric DLSs with (4,31,31) generalized symmetry in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

Remark:

for different orders N of DLSs it is well known different kinds of trees.

21. Venus

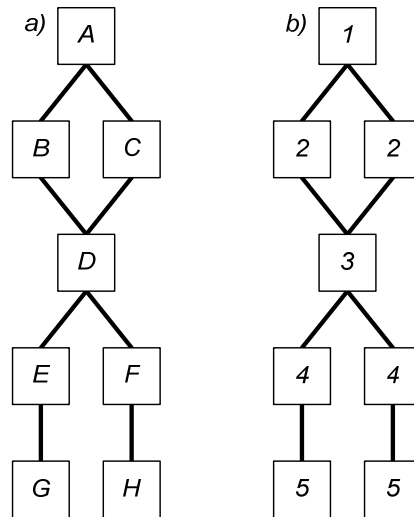


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891204789635738560149259168372043652918047907854231645902
73168846719052368413259702739064851 (DLS *A*, CF 1) – generalized
symmetry (4,31,31);
- DLS 2: 01234567895896217340304856912716720984358760123594425937160824379
05816730168495295847320616915840273 (DLS *B*, CF 2);
- DLS 3: 01234567895896217340394856012716729084358769123504425037169824370
95816730168495295847320616015849273 (DLS *C*, CF 2);
- DLS 4: 01234567891204769835736580149259182370643856910247907254831645906
73128648719250386413259702739084651 (DLS *D*, CF 3) – generalized
symmetry (4,31,31);
- DLS 5: 01234567898596217340304856912716720954385760123894428937160524379
08516730168495298547320616915840273 (DLS *E*, CF 4);
- DLS 6: 01234567898596217340394856012716729054385769123804428037169524370
98516730168495298547320616015849273 (DLS *F*, CF 4);
- DLS 7: 01234567891274069835736580149259182370643856910247970254831645906
73128648719250386413259702039784651 (DLS *G*, CF 5);
- DLS 8: 01234567891204769835786530149259182370648356910247907254831645906
73128648719250336418259702739084651 (DLS *H*, CF 5).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120478963573856014925916837204365291804790785423164590273
168846719052368413259702739064851 (DLS *A*);
CF 2: 0123456789120459863730578194269546327801591870236446701832956495231
078836107495278329651402789640513 (DLS *B*, DLS *C*);
CF 3: 0123456789120476983573658014925918237064385691024790725483164590673
128648719250386413259702739084651 (DLS *D*);
CF 4: 0123456789120459863730678194259645327801691870235445701832965496231
078835107496278329651402789640513 (DLS *E*, DLS *F*);
CF 5: 0123456789120476983578653014925918237064835691024790725483164590673
128648719250336418259702739084651 (DLS *G*, DLS *H*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 8, a = 8, \rho = [1, 1, 2, 2, 2, 2, 2, 4].$$

Method of finding:

search for generalized symmetric DLSs with (4,31,31) generalized symmetry in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

22. Daedalus-8

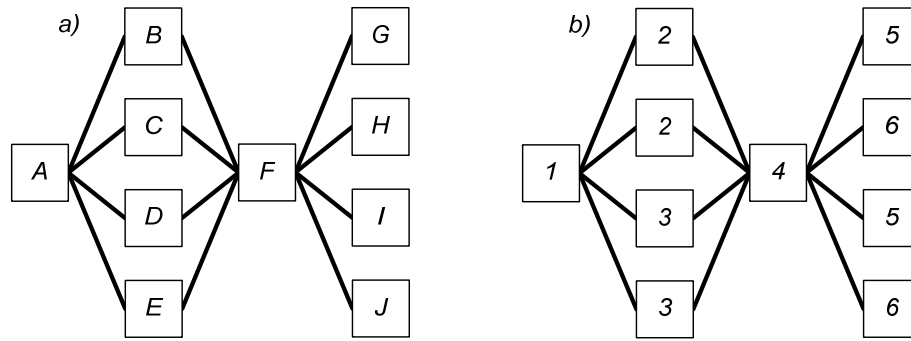


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891506793428386402519759173846024052837916279154086372806
19354637910824596482715308435962071 (DLS *A*, CF 1) – generalized
symmetry (4,31,31);
- DLS 2: 01234567895340179862647591823075968031249618542073398402165780372
64591425138790628697304151702695348 (DLS *B*, CF 2);
- DLS 3: 01234567895340179862647598123075968031249681542073391402865780372
64591425831790628697304151702695348 (DLS *C*, CF 2);
- DLS 4: 01234567895349170862647591823075068931249618542073398402165780372
64591425138790628607394151792605348 (DLS *D*, CF 3);
- DLS 5: 01234567895349170862647598123075068931249681542073391402865780372
64591425831790628607394151792605348 (DLS *E*, CF 3);
- DLS 6: 01234567891206793458386402519729173846054052837916579124086375806
19324637910854296485712308435962071 (DLS *F*, CF 4) – generalized
symmetry (4,31,31);
- DLS 7: 01234567895340179862847596123075986031249681542073391402865760372
84591425631790828697304151702895346 (DLS *G*, CF 5);
- DLS 8: 01234567895349170862847596123075086931249681542073391402865760372
84591425631790828607394151792805346 (DLS *H*, CF 6);
- DLS 9: 01234567895340179862647591823075968013249638542071198402365780172
64593425138790628697304153702695148 (DLS *I*, CF 5);
- DLS 10: 0123456789534917086264759182307506891324963854207119840236578017
264593425138790628607394153792605148 (DLS *J*, CF 6).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789123978460595672104385314602897270896531470825391464691378
052847619352068450219733950847261 (DLS *A*);
- CF 2: 0123456789120458936785926170349658734120246037895159762408137035891
642431706259868491032753781925406 (DLS *B*, DLS *C*);
- CF 3: 0123456789120437956823681974509475082136658973104239428156078731604
925465092387150172683947896540213 (DLS *D*, DLS *E*);
- CF 4: 0123456789120679345838640251972917384605405283791657912408637580619
324637910854296485712308435962071 (DLS *F*);
- CF 5: 0123456789120458936789526170345698734120246037859195762408137035891
642431706295868491032753781925406 (DLS *G*, DLS *I*);
- CF 6: 0123456789120437965824518973605689714032937062814587451309264938261
507356704289160129854737896503214 (DLS *H*, DLS *J*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 10, a = 12, \rho = [1, 1, 1, 1, 2, 2, 2, 2, 4, 8].$$

Method of finding:

search for generalized symmetric DLSs with (4,31,31) generalized symmetry in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

22. Daedalus-10

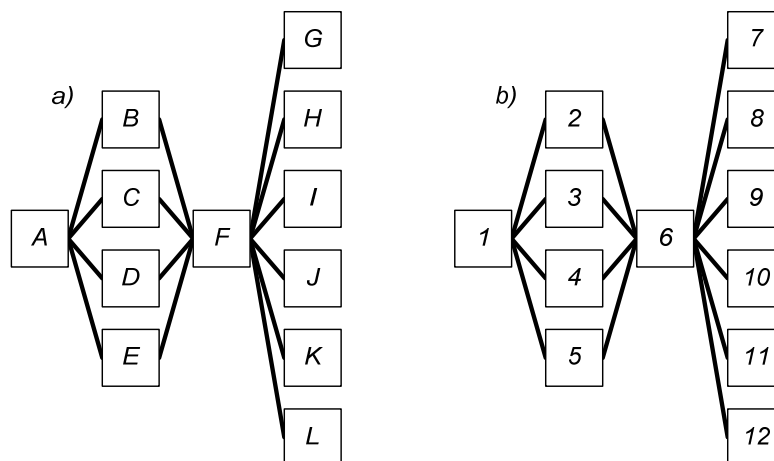


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891204537968946238517045186920373986741205275981034678302
69514867590342163910748525047128693 (DLS A, CF 1);
- DLS 2: 01234567898756092341639487105292457138602430185976598726410330716
28495486953021715029476387618309524 (DLS B, CF 2);
- DLS 3: 01234567898756092341639417805292457831602430815976591726480330786
21495486953021715029476387681309524 (DLS C, CF 3);
- DLS 4: 01234567898756092341239487105696457138206430185972598726410330716
28495486953021715029476387218309564 (DLS D, CF 4);
- DLS 5: 01234567898756092341239417805696457831206430815972591726480330786
21495486953021715029476387281309564 (DLS E, CF 5);
- DLS 6: 01234567891204537968976238514075186920343986741205245981037648302
69517867590342163910748525047128693 (DLS F, CF 6);
- DLS 7: 01234567898756092341639417805292457031682430815976591726480330786
21495486953021715829476307601389524 (DLS G, CF 7);
- DLS 8: 01234567898756092143639417805292457013682410835976593726480110786
23495486951023735829476107601389524 (DLS H, CF 8);
- DLS 9: 01234567898756092143639417805292457813602410835976593726480110786
23495486951023735029476187681309524 (DLS I, CF 9);
- DLS 10: 0123456789875609234123941780569645703128643081597259172648033078
621495486953021715829476307201389564 (DLS J, CF 10);
- DLS 11: 0123456789875609214323941780569645701328641083597259372648011078
623495486951023735829476107201389564 (DLS K, CF 11);
- DLS 12: 0123456789875609214323941780569645781320641083597259372648011078
623495486951023735029476187281309564 (DLS L, CF 12).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789120453796894623851704518692037398674120527598103467830269
514867590342163910748525047128693 (DLS A);
- CF 2: 0123456789120436597839568174024619270835806572319453421890672478901
356983754261075906382416781094523 (DLS B);
- CF 3: 0123456789120457396894867251033759168420269730485175382410968965017
234507183964263409825174812690375 (DLS C);
- CF 4: 0123456789120436597823586914077869530142641072985340928173653541078
296873590462196871425305976283014 (DLS D);
- CF 5: 0123456789120459763847869301529035642817657138942038927610457650814
293236817590484190235765947208361 (DLS E);
- CF 6: 0123456789120453796897623851407518692034398674120524598103764830269
517867590342163910748525047128693 (DLS F);
- CF 7: 0123456789120463597878395146026091728543254890316743671802955612097
834875036942194862713503975842016 (DLS G);
- CF 8: 0123456789120459863775129034689386015274463872195027591803463960274
815847136950258476320916095847123 (DLS H);
- CF 9: 0123456789120463597839568174024319270865804572319654621890372678901
354983754261075903682416781094523 (DLS I);
- CF 10: 012345678912046359782839514607609127854375489031624367180295561209
7834875036942194867213503975842016 (DLS J);
- CF 11: 012345678912045986377512309468398601527446387219502759180346936027
4815847196350258476320916095847123 (DLS K);
- CF 12: 012345678912043689579862175340534798160264195208737058634291267084
9135398501742687962035144531792068 (DLS L).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 12, a = 14, \rho = [1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 4, 10].$$

Method of finding:

search for partially central symmetric DLSs, symmetric filled cells number $M = 60$, in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

23. Robot

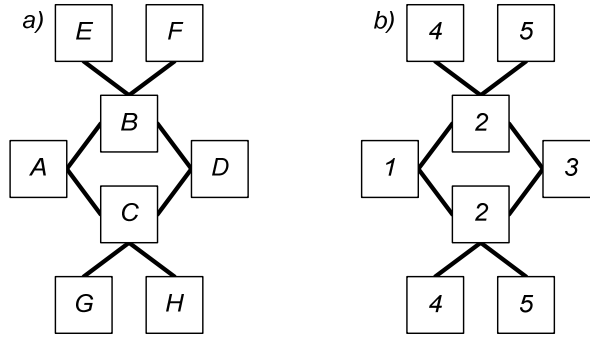


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

- DLS 1: 01234567891204785963238160459745391620787460519832967503821450983
47621694287315087162903453857921406 (DLS *A*, CF 1) – generalized
symmetry (4,31,31);
- DLS 2: 01234567898310967524489672301572645819301958642307604137589295072
18463378510924626790341585432890671 (DLS *B*, CF 2);
- DLS 3: 01234567898319067524489672301572645819301058642397694137580295072
18463378519024626709341585432809671 (DLS *C*, CF 2);
- DLS 4: 01234567891204795863238160459745391620787460518932867503921450983
47621694287315097162803453857921406 (DLS *D*, CF 3) – generalized
symmetry (4,31,31);
- DLS 5: 01234567891234785960238160459745091623787460519832967503821450983
47621694287015387162930453857921406 (DLS *E*, CF 4);
- DLS 6: 01234567891234795860238160459745091623787460518932867503921450983
47621694287015397162830453857921406 (DLS *F*, CF 5);
- DLS 7: 01234567895204781963238160459745391620787460519832967503821410983
47625694287315087562903413817925406 (DLS *G*, CF 4);
- DLS 8: 01234567895204791863238160459745391620787460518932867503921410983
47625694287315097562803413817925406 (DLS *H*, CF 5).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

- CF 1: 0123456789120478596323816045974539162078746051983296750382145098347
621694287315087162903453857921406 (DLS *A*);
- CF 2: 0123456789120657893494578326016371940528859471326036102894572968304
175784906531247856210935032197846 (DLS *B*, DLS *C*);

CF 3: 0123456789120479586323816045974539162078746051893286750392145098347
621694287315097162803453857921406 (DLS *D*);
CF 4: 0123456789123457890654926318709756803142308174926563401825978917365
024786592041345092176382678094351 (DLS *E*, DLS *G*);
CF 5: 0123456789123459786056108793247346125098685930417285972406139471638
205308576294127089134564962081537 (DLS *F*, DLS *H*).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 8, a = 8, \rho = [1, 1, 1, 1, 2, 2, 4, 4].$$

Method of finding:

search for generalized symmetric DLSs with (4,31,31) generalized symmetry in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

24. Stingray

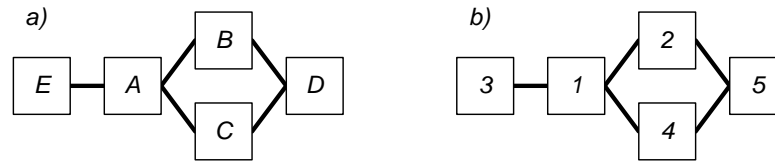


Fig. Combinatorial structure (a) and corresponding CFs (b)

DLSs within combinatorial structure:

DLS 1: 01234567891204678935583672019437859426102910865347634109752845972
81063765831940294625038718079134256 (DLS A, CF 1);

DLS 2: 01234567895492780163637190425826198730459267541830895013247617056
28394358409762140382659177846319502 (DLS B, CF 2);

DLS 3: 01234567895410789263637129405826098713453067542891895231047617356
28904958413762042980651377846903512 (DLS E, CF 3);

DLS 4: 01234567895412789063637109425826098713453267540891895031247617356
28904958413762040982651377846903512 (DLS C, CF 4);

DLS 5: 01234567893204678915581672039417859426302930865147634109752845972
83061765831940294625018738079134256 (DLS D, CF 5).

Adjacency matrix:

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}.$$

Different CFs set within combinatorial structure:

CF 1: 0123456789120467893558367201943785942610291086534763410975284597281
063765831940294625038718079134256 (DLS A);

CF 2: 0123456789123486905760475928138659730124539017864229056843719582317
460471602359874682019353871945206 (DLS B);

CF 3: 0123456789120458963737810245964675931208253867091473608954219452317
860894620317550197683426897142053 (DLS E);

CF 4: 0123456789120458936767850214939547612830246837091576308945214371965
208895620317450197386423892147056 (DLS C);

CF 5: 0123456789120467985359807236146795140238985631742070198653423567284
901463809217584725310962341908567 (DLS D).

Number of vertices, number of edges, ascending sorted vector of vertices powers:

$$v = 5, a = 5, \rho = [1, 2, 2, 2, 3].$$

Method of finding:

search for partially central symmetric DLSs, symmetric filled cells number $M = 80$, in combination with method of checking DLS for ODLS (Euler-Parker method, DLX).

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