

Southwest State University
Department of computer sciences

**USING CELL MAPPING SCHEMES
DURING SEARCHING FOR PAIRS
OF ORTHOGONAL DIAGONAL
LATIN SQUARES**

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Pereslavl-Zalessky, 2019



What is Latin squares?

$$A = \left\| a_{ij} \right\|$$

$$i, j = \overline{1, N}$$

$$N = |S|$$

$$S = \{0, 1, 2, \dots, N-1\}$$

$$\forall i, j, k = \overline{1, N}, j \neq k : (a_{ij} \neq a_{ik}) \wedge (a_{ji} \neq a_{ki})$$

$$\forall i, j = \overline{1, N}, i \neq j : (a_{ii} \neq a_{jj}) \wedge (a_{N-i+1, N-i+1} \neq a_{N-j+1, N-j+1})$$

0	1	2	3	4	5	6	7	8	9
1	2	9	4	3	6	7	5	0	8
2	9	3	1	7	0	5	8	4	6
3	4	1	2	8	7	9	6	5	0
4	3	5	9	2	1	8	0	6	7
5	6	4	8	1	2	0	9	7	3
6	5	8	7	0	3	2	1	9	4
7	8	6	0	9	4	1	2	3	5
8	7	0	5	6	9	3	4	1	2
9	0	7	6	5	8	4	3	2	1

Normalized LS of order 10

$$N! \times (N-1)!$$

0	1	2	3	4	5	6	7	8	9
7	2	4	9	0	6	5	1	3	8
8	3	6	7	5	9	0	2	4	1
2	6	8	5	1	7	4	0	9	3
5	8	9	1	7	0	3	4	6	2
9	4	1	2	8	3	7	6	0	5
4	7	5	6	9	1	8	3	2	0
3	0	7	8	2	4	1	9	5	6
6	5	0	4	3	2	9	8	1	7
1	9	3	0	6	8	2	5	7	4

Normalized DLS of order 10

$$(N-1)!$$



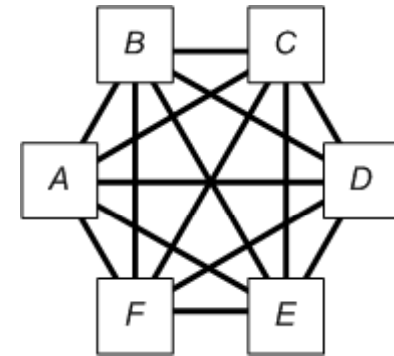
Why is this interesting?

Applied problems:

- experiment planning
- cryptography
- error correcting codes
- scheduling
- algebra, combinatorics, statistics, ...

Mathematical problems:

- **existence of a triple of MOLS/MODLS of order 10 (or larger clique)**
- increasing world record of orthogonality characteristic for pseudo triple of MOLS (291/300) or MODLS (274/300)
- generating functions
- asymptotic behavior of combinatorial characteristics based on DLSs (OEIS)
- number theory (relations between different fields of knowledge)
- magic squares
- Sudoku (LS of order 9 with additional constraints)



Brief history and approaches

- Euler — triple of MOLS of order 10 does not exist (**disproved**);
- Parker et al. (1960) — **first pair of MOLS** of order 10 using transversals;
- Brown et al. (1995) — horizontally symmetric DLS, **1:4**;
- Zaikin et al. (2015—2016) — SAT approach (system of Boolean equations), ~100 CFs of ODLS, **onces**;
- Vatutin et al. (from 2017) — **RS+LBF generator -> transversals + DLX**, ~1M CFs of ODLS, **twices, 1:3**;
- Vatutin et al. (2017) — **plane symmetry generator -> transversals + DLX**, ~200k CFs of ODLS, **1:6, 1:8, rhombus-4, line-4, line-5, loop-4, fish**;
- Vatutin et al. (from 2017) — **central symmetry, partial central and plane symmetries, generalized symmetries, neighborhoods of generalized symmetris generator -> canonizer -> postprocessor**, ~7M CFs of ODLS, **1:5, 1:7, 1:10, rhombus-3, cross, flyer, tree-1, Venus, Daedalus-8, Daedalus-10, robot, stingray**;
- Vatutin et al. (from 2019) — **none transversal search for ODLS**.



Classical search for pairs of ODLS of order 10



L. Euler expected that for $N=10$ ODLS doesn't exist
 First pair — Parker et al., 1960

0	1	2	3	4	5	6	7	8	9
1	2	0	4	3	7	9	8	5	6
7	3	5	9	0	4	8	6	2	1
3	5	6	8	9	0	4	1	7	2
4	9	7	2	6	8	1	5	0	3
5	8	4	6	7	1	3	2	9	0
8	4	9	1	2	3	7	0	6	5
6	7	3	0	1	2	5	9	4	8
9	0	1	5	8	6	2	4	3	7
2	6	8	7	5	9	0	3	1	4

0	1	2	3	4	5	6	7	8	9
7	5	1	9	2	8	0	4	6	3
1	0	3	4	6	7	5	2	9	8
9	8	4	7	5	2	1	0	3	6
6	7	9	0	8	3	2	1	5	4
4	6	5	1	0	9	8	3	2	7
2	3	8	5	1	6	4	9	7	0
5	2	7	8	3	4	9	6	0	1
3	4	6	2	9	0	7	8	1	5
8	9	0	6	7	1	3	5	4	2

SAT@Home, 04.2015



0	1	2	3	4	5	6	7	8	9
4	9	0	8	5	6	3	1	2	7
2	5	7	9	6	4	0	8	1	3
9	0	4	6	8	7	1	5	3	2
6	7	5	2	1	3	8	0	9	4
1	8	3	5	7	2	9	6	4	0
7	3	1	0	9	8	4	2	6	5
8	2	6	4	0	9	5	3	7	1
3	4	8	1	2	0	7	9	5	6
5	6	9	7	3	1	2	4	0	8

0	1	2	3	4	5	6	7	8	9
6	5	9	7	0	8	2	3	1	4
4	7	1	2	3	9	8	0	6	5
1	2	0	4	5	3	7	6	9	8
2	6	8	0	9	4	1	5	3	7
8	4	6	9	2	7	0	1	5	3
5	0	4	6	8	2	3	9	7	1
9	3	5	1	7	6	4	8	0	2
7	8	3	5	6	1	9	4	2	0
3	9	7	8	1	0	5	2	4	6

Gerasim@Home, 04.2017

Very rare combinatorial objects:
~ 30 millions DLS of order 10
 has only 1 pair of ODLS!



Searching for ODLs: approaches

- Brute Force + backtracking + clippings + ordering + ... (very long)
- SAT (some tens of hours, long)
- filling by pairs of elements $[a_{ij}, b_{ij}]$ (long)
- using transversals (fast) – **200 – 800 DLS/s** for different algorithms!
- using transversals with canonizer (**~8000 DLS/s** effective pace)

a)

0	1	2	3	4
4	2	3	0	1
3	4	1	2	0
1	3	0	4	2
2	0	4	1	3

b)

0				
				1
			2	
	3			
		4		

$$T^{(d)}_1 = \{a_{11}, a_{25}, a_{34}, a_{42}, a_{53}\}$$

c)

	1			
		3		
				0
			4	
2				

$$T^{(d)}_2 = \{a_{12}, a_{23}, a_{35}, a_{43}, a_{51}\}$$

d)

		2		
			0	
	4			
1				
				3

$$T^{(d)}_3 = \{a_{13}, a_{24}, a_{32}, a_{41}, a_{55}\}$$

e)

			3	
4				
		1		
				2
	0			

$$T^{(d)}_4 = \{a_{14}, a_{21}, a_{33}, a_{45}, a_{52}\}$$

f)

				4
	2			
3				
		0		
				1

$$T^{(d)}_5 = \{a_{15}, a_{22}, a_{31}, a_{43}, a_{54}\}$$



Properties of cliques within combinatorial structures of small order (N=4)



- clique-2, 1 CF



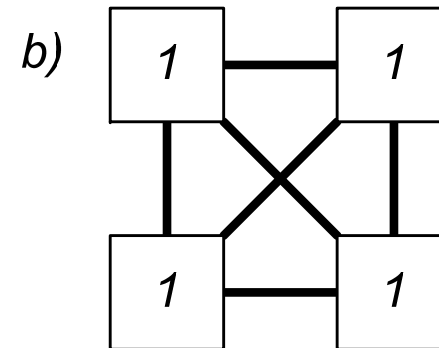
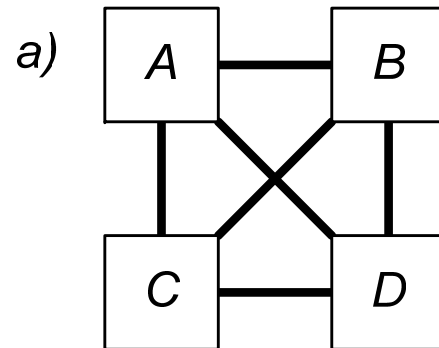
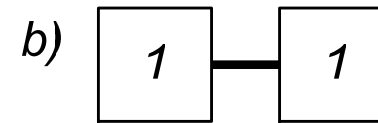
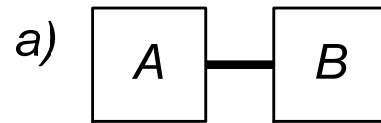
Properties of cliques within combinatorial structures of small order (N=5)



- clique-2, 1 CF



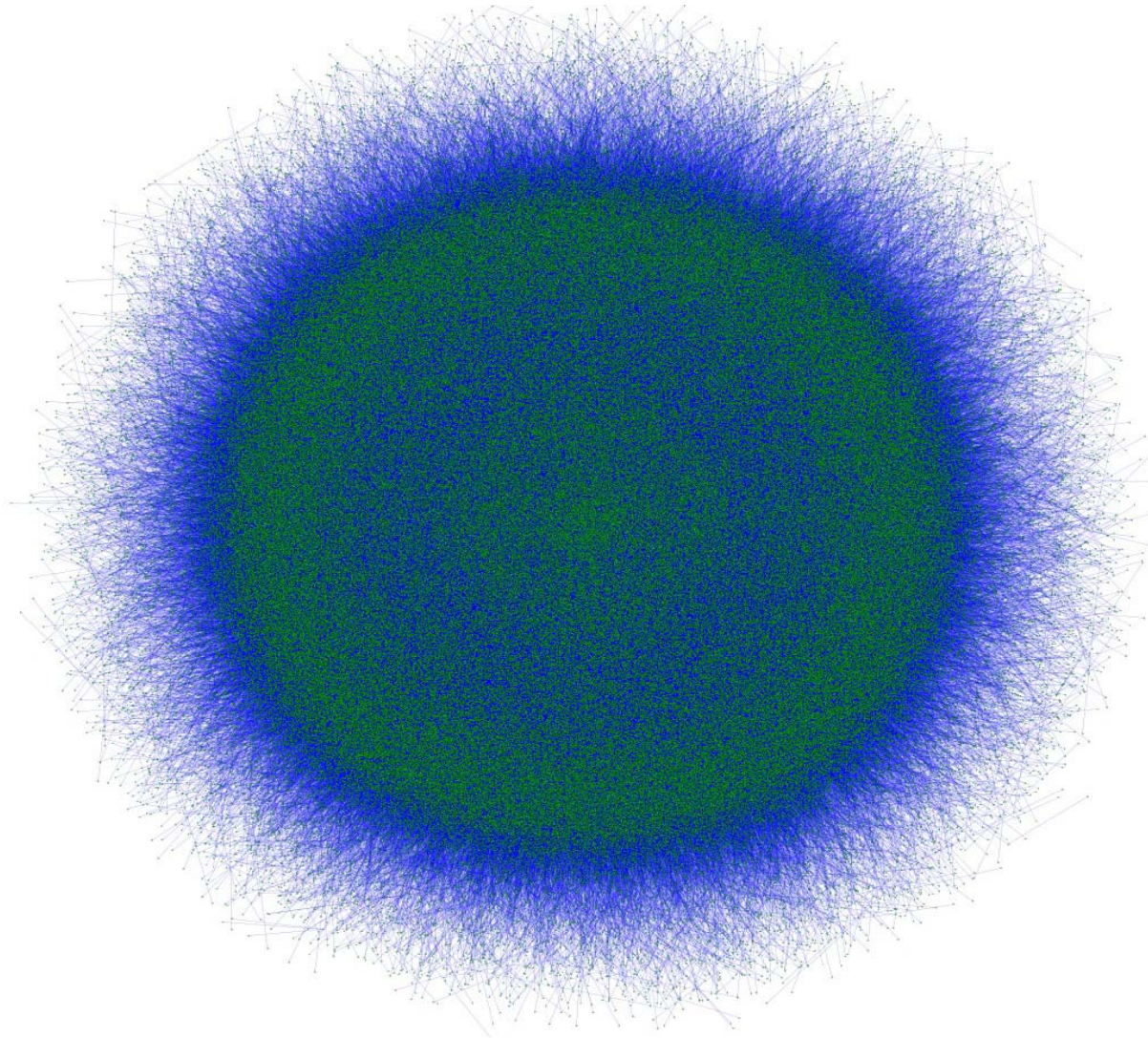
Properties of cliques within combinatorial structures of small order (N=7)



- clique-2, 1 CF
- clique-4, 1 CF



Properties of cliques within combinatorial structures of small order (N=8)



- N824HUGE structure (348000 DLSs, 657 CFs), includes clique-6, **1 CF**

Properties of cliques within combinatorial structures of small order (N=9)

197 different combinatorial structures (some of them with cliques):

- 24N54M4C — clique-3, 1 CF;
- 120N480M5C — clique-4, 1 CF;
- 24N60M4C — clique-4, 1 CF;
- 32N86M13C — **clique-3, 3 CFs**;
- 48N126M6C — clique-6, **2 CFs**.

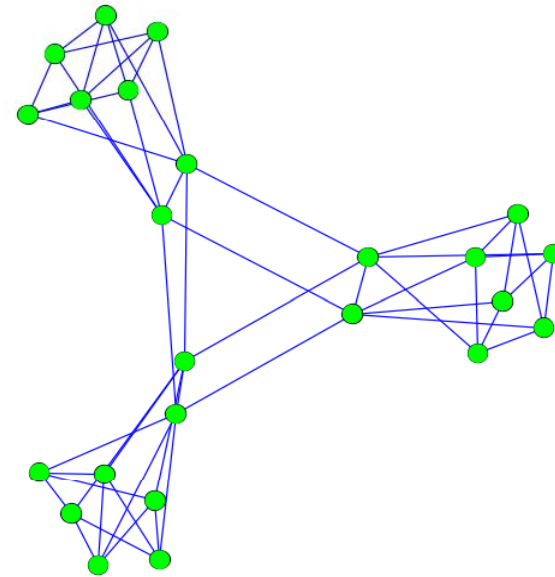


Figure 148. Graph of ODLS from workunit R9_000463421/02.
2017.12.15. Rake Search project. [B@P] Daniel (BOINC@Poland) and LCB001 (Hardware Canucks)

[http://evatutin.narod.ru/evatutin Is all structs n1to8 eng.pdf](http://evatutin.narod.ru/evatutin%20Is%20all%20structs%20n1to8%20eng.pdf)

[http://evatutin.narod.ru/evatutin Is all structs n9 eng.pdf](http://evatutin.narod.ru/evatutin%20Is%20all%20structs%20n9%20eng.pdf)

[http://evatutin.narod.ru/evatutin Is all structs eng.pdf](http://evatutin.narod.ru/evatutin%20Is%20all%20structs%20eng.pdf)

- based on RakeSearch project results (<https://rake.boincfast.ru/rakesearch/>)



Properties of cliques within combinatorial structures of order $N=10$

More than clique-2 subgraphs don't known...

Searching for ODLS from same main class?

0	1	2	3	4	5	6	7	8	9
5	9	6	4	8	1	3	0	2	7
9	0	1	8	6	2	7	4	5	3
4	6	5	2	0	7	8	3	9	1
2	4	9	7	3	6	1	8	0	5
3	7	8	9	5	4	0	2	1	6
7	8	3	0	2	9	5	1	6	4
8	5	7	1	9	0	4	6	3	2
6	3	4	5	1	8	2	9	7	0
1	2	0	6	7	3	9	5	4	8

SODLS

- Bennett F.E., Beiliang Du, Hantao Zhang. Existence of self-orthogonal diagonal Latin squares with a missing subsquare // Discrete Mathematics. Vol. 261. 2003. pp. 69-86.



SODLS can be extended for ESODLS

This site is supported by donations to [The OEIS Foundation](#).



founded in 1964 by N. J. A. Sloane

ESODLS (Ed's SODLS) — MODLS from DLSs within same main class

OEIS sequences (SODLS, H. White):

- A287761 — 1, 0, 0, 2, 4, 0, 64, 1152, 224832;
- A287762 — 1, 0, 0, 48, 480, 0, 322560, 46448640, 81587036160.

OEIS sequences (ESODLS, new):

- A309210 — 1, 0, 0, 1, 1, 0, 5, 23;
- A309598 — 1, 0, 0, 2, 4, 0, 256, 4608;
- A309599 — 1, 0, 0, 48, 480, 0, 1290240, 185794560.

 Search [Hints](#)
 (Greetings from [The On-Line Encyclopedia of Integer Sequences!](#))

A309598 Number of extended self-orthogonal diagonal Latin squares of order n with ordered first string. 0

1, 0, 0, 2, 4, 0, 256, 4608 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

OFFSET 1,4

COMMENTS A self-orthogonal diagonal Latin square (SODLS) is a diagonal Latin square orthogonal to its transpose. An extended self-orthogonal diagonal Latin square (ESODLS) is a diagonal Latin square that has an orthogonal diagonal Latin square from the same main class. SODLS is a special case of ESODLS.

LINKS [Table of n, a\(n\) for n=1..8.](#)
E. I. Vatutin, [Discussion about properties of diagonal Latin squares](#) (in Russian)
[Index entries for sequences related to Latin squares and rectangles](#)

EXAMPLE The diagonal Latin square

```

0 1 2 3 4 5 6 7 8 9
1 2 0 4 5 7 9 8 6 3
5 0 1 6 3 9 8 2 4 7
9 3 5 8 2 1 7 4 0 6
4 6 3 5 7 8 0 9 2 1
8 4 6 9 1 3 2 5 7 0
7 8 9 0 6 4 5 1 3 2
2 9 4 7 8 0 3 6 1 5
6 5 7 1 0 2 4 3 9 8
3 7 8 2 9 6 1 0 5 4
    
```

has orthogonal diagonal Latin square

```

0 1 2 3 4 5 6 7 8 9
3 5 9 8 6 2 0 1 4 7
4 3 8 7 2 1 9 0 5 6
6 9 3 4 8 0 1 2 7 5
7 2 0 1 9 3 5 8 6 4
2 0 1 5 7 6 4 9 3 8
8 6 4 2 0 9 7 5 1 3
1 7 6 0 5 4 8 3 9 2
9 8 5 6 1 7 3 4 2 0
5 4 7 9 3 8 2 6 0 1
    
```

from the same main class.

CROSSREFS Cf. [A287761](#).

Sequence in context: [A287761](#) [A009512](#) [A317411](#) * [A305570](#) [A287651](#) [A163259](#)
Adjacent sequences: [A309595](#) [A309596](#) [A309597](#) * [A309599](#) [A309600](#) [A309601](#)

KEYWORD nonn,more

AUTHOR [Eduard I. Vatutin](#), Aug 09 2019

STATUS approved

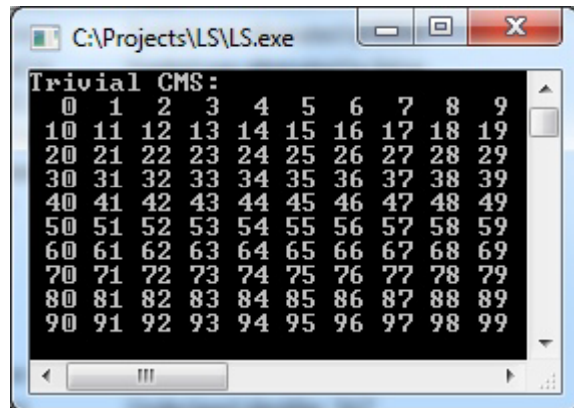


How we can find ESODLS? CMS-based search...

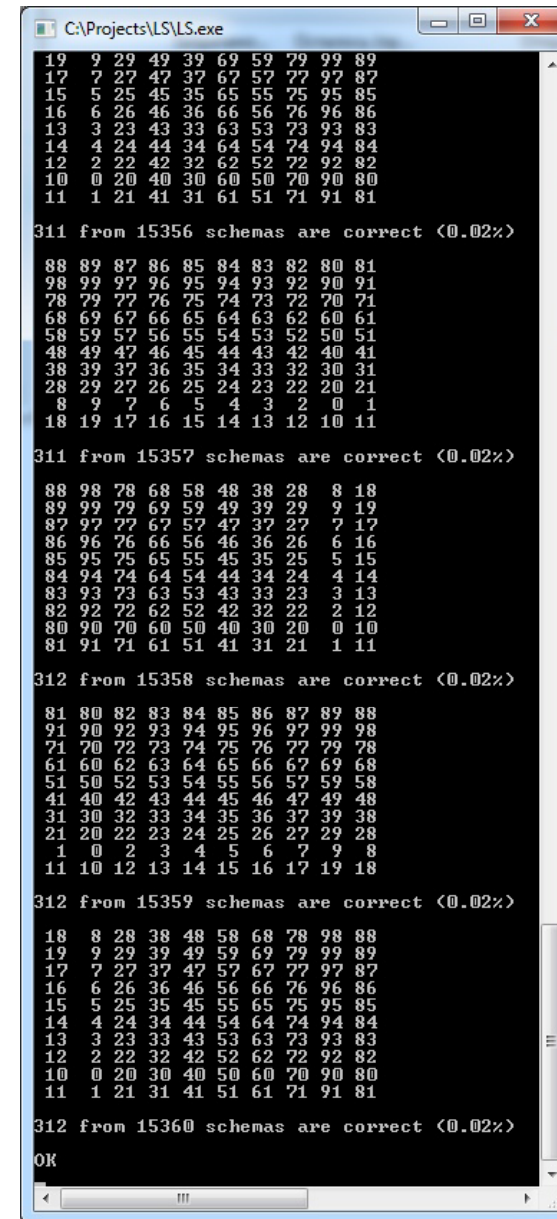
Bijjective mapping for N^2 cells of square with some special properties

SODLS — one of them...

M-transformations
(15 360 combinations for $N=10$)



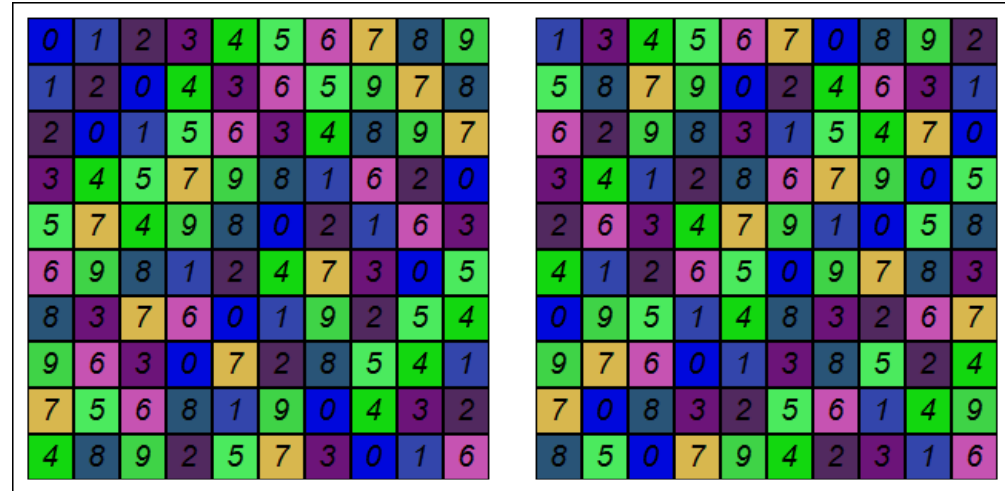
```
C:\Projects\LS\LS.exe
Trivial CMS:
0 1 2 3 4 5 6 7 8 9
10 11 12 13 14 15 16 17 18 19
20 21 22 23 24 25 26 27 28 29
30 31 32 33 34 35 36 37 38 39
40 41 42 43 44 45 46 47 48 49
50 51 52 53 54 55 56 57 58 59
60 61 62 63 64 65 66 67 68 69
70 71 72 73 74 75 76 77 78 79
80 81 82 83 84 85 86 87 88 89
90 91 92 93 94 95 96 97 98 99
```



```
C:\Projects\LS\LS.exe
19 9 29 49 39 69 59 79 99 89
17 7 27 47 37 67 57 77 97 87
15 5 25 45 35 65 55 75 95 85
16 6 26 46 36 66 56 76 96 86
13 3 23 43 33 63 53 73 93 83
14 4 24 44 34 64 54 74 94 84
12 2 22 42 32 62 52 72 92 82
10 0 20 40 30 60 50 70 90 80
11 1 21 41 31 61 51 71 91 81
311 from 15356 schemas are correct (0.02%)
88 89 87 86 85 84 83 82 80 81
98 99 97 96 95 94 93 92 90 91
78 79 77 76 75 74 73 72 70 71
68 69 67 66 65 64 63 62 60 61
58 59 57 56 55 54 53 52 50 51
48 49 47 46 45 44 43 42 40 41
38 39 37 36 35 34 33 32 30 31
28 29 27 26 25 24 23 22 20 21
8 9 7 6 5 4 3 2 0 1
18 19 17 16 15 14 13 12 10 11
311 from 15357 schemas are correct (0.02%)
88 98 78 68 58 48 38 28 8 18
89 99 79 69 59 49 39 29 9 19
87 97 77 67 57 47 37 27 7 17
86 96 76 66 56 46 36 26 6 16
85 95 75 65 55 45 35 25 5 15
84 94 74 64 54 44 34 24 4 14
83 93 73 63 53 43 33 23 3 13
82 92 72 62 52 42 32 22 2 12
80 90 70 60 50 40 30 20 0 10
81 91 71 61 51 41 31 21 1 11
312 from 15358 schemas are correct (0.02%)
81 80 82 83 84 85 86 87 89 88
91 90 92 93 94 95 96 97 99 98
71 70 72 73 74 75 76 77 79 78
61 60 62 63 64 65 66 67 69 68
51 50 52 53 54 55 56 57 59 58
41 40 42 43 44 45 46 47 49 48
31 30 32 33 34 35 36 37 39 38
21 20 22 23 24 25 26 27 29 28
1 0 2 3 4 5 6 7 9 8
11 10 12 13 14 15 16 17 19 18
312 from 15359 schemas are correct (0.02%)
18 8 28 38 48 58 68 78 98 88
19 9 29 39 49 59 69 79 99 89
17 7 27 37 47 57 67 77 97 87
16 6 26 36 46 56 66 76 96 86
15 5 25 35 45 55 65 75 95 85
14 4 24 34 44 54 64 74 94 84
13 3 23 33 43 53 63 73 93 83
12 2 22 32 42 52 62 72 92 82
10 0 20 30 40 50 60 70 90 80
11 1 21 31 41 51 61 71 91 81
312 from 15360 schemas are correct (0.02%)
OK
```

Different properties of CMS: example 1

CMS[0] = 0	CMS[34] = 33	CMS[68] = 69
CMS[1] = 1	CMS[35] = 13	CMS[69] = 68
CMS[2] = 2	CMS[36] = 59	CMS[70] = 52
CMS[3] = 3	CMS[37] = 60	CMS[71] = 81
CMS[4] = 4	CMS[38] = 50	CMS[72] = 26
CMS[5] = 5	CMS[39] = 25	CMS[73] = 63
CMS[6] = 6	CMS[40] = 28	CMS[74] = 58
CMS[7] = 7	CMS[41] = 90	CMS[75] = 79
CMS[8] = 8	CMS[42] = 53	CMS[76] = 56
CMS[9] = 9	CMS[43] = 67	CMS[77] = 57
CMS[10] = 30	CMS[44] = 94	CMS[78] = 17
CMS[11] = 95	CMS[45] = 19	CMS[79] = 75
CMS[12] = 32	CMS[46] = 64	CMS[80] = 23
CMS[13] = 35	CMS[47] = 24	CMS[81] = 71
CMS[14] = 48	CMS[48] = 14	CMS[82] = 29
CMS[15] = 92	CMS[49] = 62	CMS[83] = 22
CMS[16] = 93	CMS[50] = 38	CMS[84] = 66
CMS[17] = 78	CMS[51] = 21	CMS[85] = 96
CMS[18] = 65	CMS[52] = 70	CMS[86] = 87
CMS[19] = 45	CMS[53] = 42	CMS[87] = 86
CMS[20] = 31	CMS[54] = 88	CMS[88] = 54
CMS[21] = 51	CMS[55] = 99	CMS[89] = 27
CMS[22] = 83	CMS[56] = 76	CMS[90] = 41
CMS[23] = 80	CMS[57] = 77	CMS[91] = 61
CMS[24] = 47	CMS[58] = 74	CMS[92] = 15
CMS[25] = 39	CMS[59] = 36	CMS[93] = 16
CMS[26] = 72	CMS[60] = 37	CMS[94] = 44
CMS[27] = 89	CMS[61] = 91	CMS[95] = 11
CMS[28] = 40	CMS[62] = 49	CMS[96] = 85
CMS[29] = 82	CMS[63] = 73	CMS[97] = 98
CMS[30] = 10	CMS[64] = 46	CMS[98] = 97
CMS[31] = 20	CMS[65] = 18	CMS[99] = 55
CMS[32] = 12	CMS[66] = 84	
CMS[33] = 34	CMS[67] = 43	

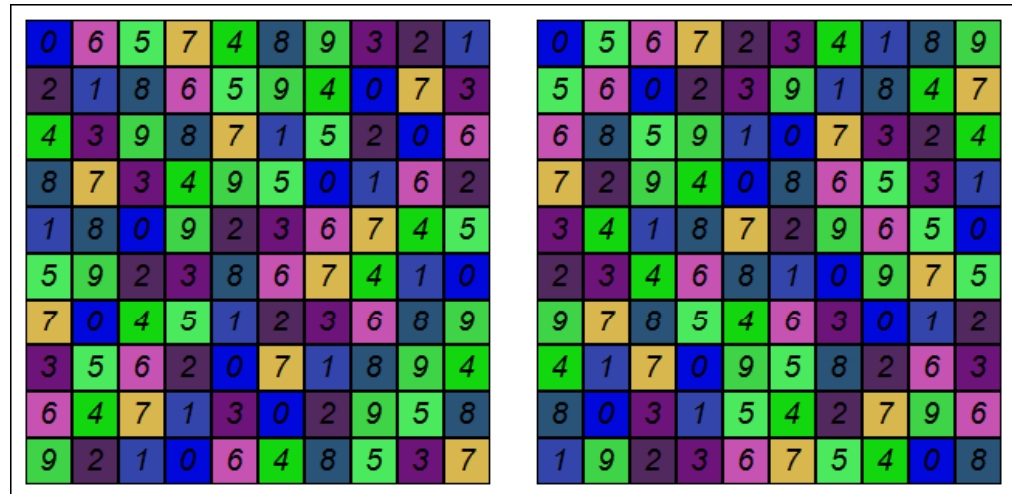


82 hours per ODLS pair
(~10 times slower)



Different properties of CMS: example 2

CMS[0] = 0	CMS[34] = 42	CMS[68] = 78
CMS[1] = 44	CMS[35] = 45	CMS[69] = 71
CMS[2] = 77	CMS[36] = 12	CMS[70] = 62
CMS[3] = 38	CMS[37] = 91	CMS[71] = 69
CMS[4] = 97	CMS[38] = 3	CMS[72] = 72
CMS[5] = 47	CMS[39] = 22	CMS[73] = 59
CMS[6] = 16	CMS[40] = 60	CMS[74] = 25
CMS[7] = 43	CMS[41] = 20	CMS[75] = 27
CMS[8] = 84	CMS[42] = 34	CMS[76] = 32
CMS[9] = 9	CMS[43] = 7	CMS[77] = 2
CMS[10] = 10	CMS[44] = 1	CMS[78] = 68
CMS[11] = 23	CMS[45] = 35	CMS[79] = 18
CMS[12] = 36	CMS[46] = 58	CMS[80] = 19
CMS[13] = 26	CMS[47] = 5	CMS[81] = 85
CMS[14] = 31	CMS[48] = 52	CMS[82] = 82
CMS[15] = 83	CMS[49] = 28	CMS[83] = 15
CMS[16] = 6	CMS[50] = 50	CMS[84] = 8
CMS[17] = 98	CMS[51] = 24	CMS[85] = 81
CMS[18] = 79	CMS[52] = 48	CMS[86] = 63
CMS[19] = 80	CMS[53] = 54	CMS[87] = 55
CMS[20] = 41	CMS[54] = 53	CMS[88] = 92
CMS[21] = 21	CMS[55] = 87	CMS[89] = 89
CMS[22] = 39	CMS[56] = 93	CMS[90] = 90
CMS[23] = 11	CMS[57] = 64	CMS[91] = 37
CMS[24] = 51	CMS[58] = 46	CMS[92] = 88
CMS[25] = 74	CMS[59] = 73	CMS[93] = 56
CMS[26] = 13	CMS[60] = 40	CMS[94] = 30
CMS[27] = 75	CMS[61] = 67	CMS[95] = 29
CMS[28] = 49	CMS[62] = 70	CMS[96] = 65
CMS[29] = 95	CMS[63] = 86	CMS[97] = 4
CMS[30] = 94	CMS[64] = 57	CMS[98] = 17
CMS[31] = 14	CMS[65] = 96	CMS[99] = 66
CMS[32] = 76	CMS[66] = 99	
CMS[33] = 33	CMS[67] = 61	



28 seconds per ODLS pair
 (~1000 times faster)

- Nested loops implementation?
- GPU/Phi implementation?



What we know about ESODLS of order 10?

- 32010 ESODLS CFs — Gerasim@Home results:
 - 30429 SODLS CFs — SOLS to SODLS (whitefox);
 - 1581 CFs — Gerasim@Home generalized symmetries neighborhoods.

Combinatorial structures:

ONCE (A):1 - 32010, where:
1 CFs - 32010

LINE4 (C):1 - 3, where:
2 CFs - 3

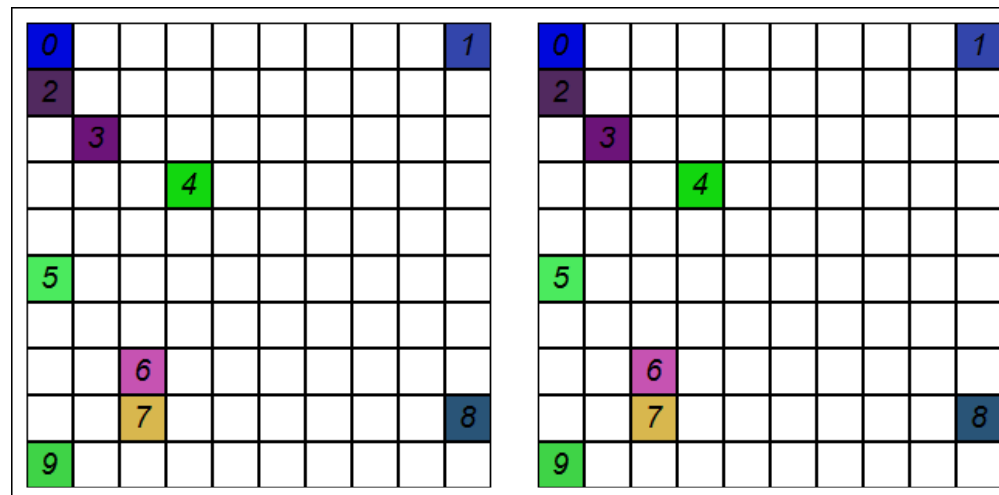
LINE4 (C):2 - 3, where:
2 CFs - 3

LOOP4 (E):2 - 76, where:
1 CFs - 2
2 CFs - 74



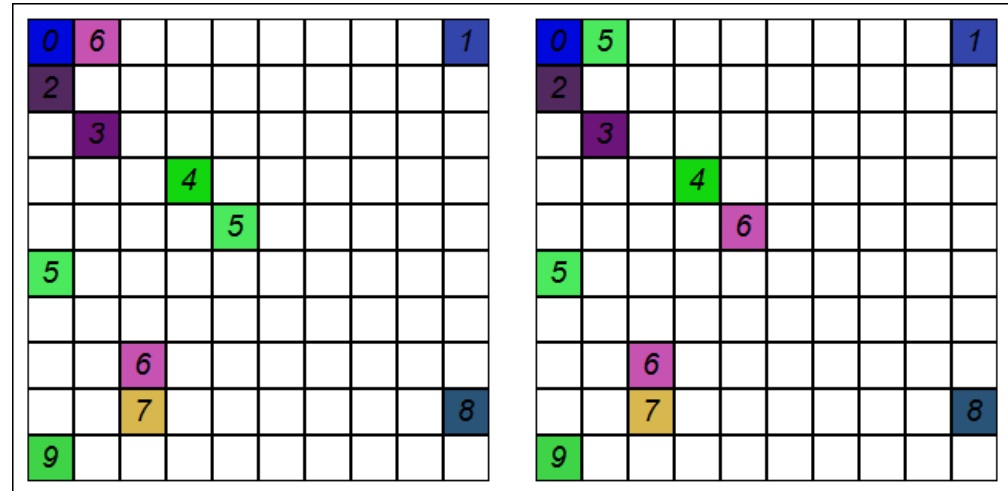
CMS filling example: fixed points

- | | | |
|---------------------|---------------------|---------------------|
| CMS[0] = 0 | CMS[34] = 42 | CMS[68] = 78 |
| CMS[1] = 44 | CMS[35] = 45 | CMS[69] = 71 |
| CMS[2] = 77 | CMS[36] = 12 | CMS[70] = 62 |
| CMS[3] = 38 | CMS[37] = 91 | CMS[71] = 69 |
| CMS[4] = 97 | CMS[38] = 3 | CMS[72] = 72 |
| CMS[5] = 47 | CMS[39] = 22 | CMS[73] = 59 |
| CMS[6] = 16 | CMS[40] = 60 | CMS[74] = 25 |
| CMS[7] = 43 | CMS[41] = 20 | CMS[75] = 27 |
| CMS[8] = 84 | CMS[42] = 34 | CMS[76] = 32 |
| CMS[9] = 9 | CMS[43] = 7 | CMS[77] = 2 |
| CMS[10] = 10 | CMS[44] = 1 | CMS[78] = 68 |
| CMS[11] = 23 | CMS[45] = 35 | CMS[79] = 18 |
| CMS[12] = 36 | CMS[46] = 58 | CMS[80] = 19 |
| CMS[13] = 26 | CMS[47] = 5 | CMS[81] = 85 |
| CMS[14] = 31 | CMS[48] = 52 | CMS[82] = 82 |
| CMS[15] = 83 | CMS[49] = 28 | CMS[83] = 15 |
| CMS[16] = 6 | CMS[50] = 50 | CMS[84] = 8 |
| CMS[17] = 98 | CMS[51] = 24 | CMS[85] = 81 |
| CMS[18] = 79 | CMS[52] = 48 | CMS[86] = 63 |
| CMS[19] = 80 | CMS[53] = 54 | CMS[87] = 55 |
| CMS[20] = 41 | CMS[54] = 53 | CMS[88] = 92 |
| CMS[21] = 21 | CMS[55] = 87 | CMS[89] = 89 |
| CMS[22] = 39 | CMS[56] = 93 | CMS[90] = 90 |
| CMS[23] = 11 | CMS[57] = 64 | CMS[91] = 37 |
| CMS[24] = 51 | CMS[58] = 46 | CMS[92] = 88 |
| CMS[25] = 74 | CMS[59] = 73 | CMS[93] = 56 |
| CMS[26] = 13 | CMS[60] = 40 | CMS[94] = 30 |
| CMS[27] = 75 | CMS[61] = 67 | CMS[95] = 29 |
| CMS[28] = 49 | CMS[62] = 70 | CMS[96] = 65 |
| CMS[29] = 95 | CMS[63] = 86 | CMS[97] = 4 |
| CMS[30] = 94 | CMS[64] = 57 | CMS[98] = 17 |
| CMS[31] = 14 | CMS[65] = 96 | CMS[99] = 66 |
| CMS[32] = 76 | CMS[66] = 99 | |
| CMS[33] = 33 | CMS[67] = 61 | |



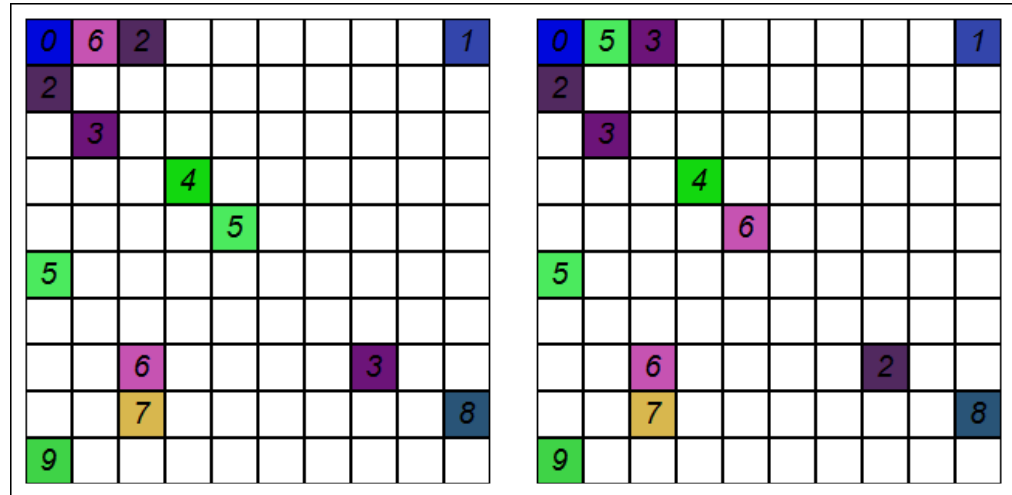
CMS filling example: fixed points + 1st pair

- | | | |
|---------------------|---------------------|---------------------|
| CMS[0] = 0 | CMS[34] = 42 | CMS[68] = 78 |
| CMS[1] = 44 | CMS[35] = 45 | CMS[69] = 71 |
| CMS[2] = 77 | CMS[36] = 12 | CMS[70] = 62 |
| CMS[3] = 38 | CMS[37] = 91 | CMS[71] = 69 |
| CMS[4] = 97 | CMS[38] = 3 | CMS[72] = 72 |
| CMS[5] = 47 | CMS[39] = 22 | CMS[73] = 59 |
| CMS[6] = 16 | CMS[40] = 60 | CMS[74] = 25 |
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| CMS[8] = 84 | CMS[42] = 34 | CMS[76] = 32 |
| CMS[9] = 9 | CMS[43] = 7 | CMS[77] = 2 |
| CMS[10] = 10 | CMS[44] = 1 | CMS[78] = 68 |
| CMS[11] = 23 | CMS[45] = 35 | CMS[79] = 18 |
| CMS[12] = 36 | CMS[46] = 58 | CMS[80] = 19 |
| CMS[13] = 26 | CMS[47] = 5 | CMS[81] = 85 |
| CMS[14] = 31 | CMS[48] = 52 | CMS[82] = 82 |
| CMS[15] = 83 | CMS[49] = 28 | CMS[83] = 15 |
| CMS[16] = 6 | CMS[50] = 50 | CMS[84] = 8 |
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| CMS[19] = 80 | CMS[53] = 54 | CMS[87] = 55 |
| CMS[20] = 41 | CMS[54] = 53 | CMS[88] = 92 |
| CMS[21] = 21 | CMS[55] = 87 | CMS[89] = 89 |
| CMS[22] = 39 | CMS[56] = 93 | CMS[90] = 90 |
| CMS[23] = 11 | CMS[57] = 64 | CMS[91] = 37 |
| CMS[24] = 51 | CMS[58] = 46 | CMS[92] = 88 |
| CMS[25] = 74 | CMS[59] = 73 | CMS[93] = 56 |
| CMS[26] = 13 | CMS[60] = 40 | CMS[94] = 30 |
| CMS[27] = 75 | CMS[61] = 67 | CMS[95] = 29 |
| CMS[28] = 49 | CMS[62] = 70 | CMS[96] = 65 |
| CMS[29] = 95 | CMS[63] = 86 | CMS[97] = 4 |
| CMS[30] = 94 | CMS[64] = 57 | CMS[98] = 17 |
| CMS[31] = 14 | CMS[65] = 96 | CMS[99] = 66 |
| CMS[32] = 76 | CMS[66] = 99 | |
| CMS[33] = 33 | CMS[67] = 61 | |



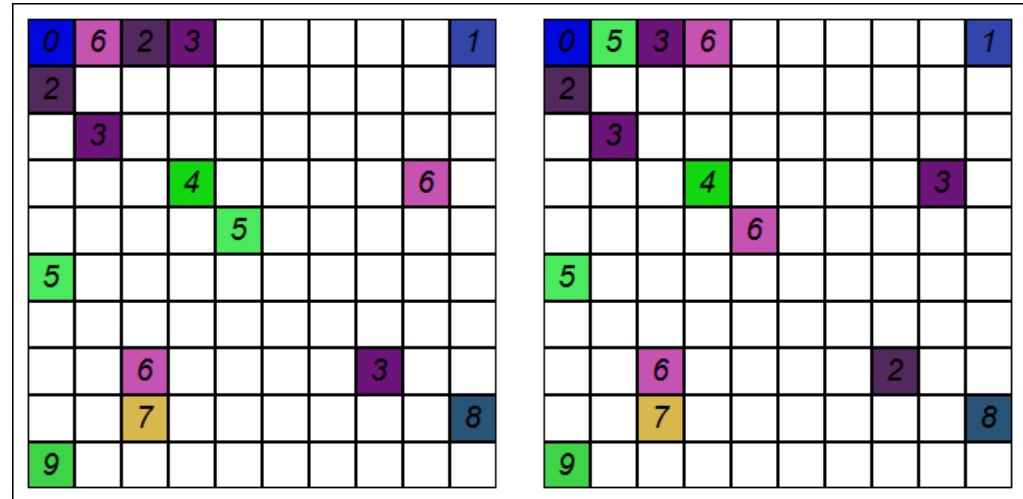
CMS filling example: fixed points + 2nd pairs

- | | | |
|---------------------|---------------------|---------------------|
| CMS[0] = 0 | CMS[34] = 42 | CMS[68] = 78 |
| CMS[1] = 44 | CMS[35] = 45 | CMS[69] = 71 |
| CMS[2] = 77 | CMS[36] = 12 | CMS[70] = 62 |
| CMS[3] = 38 | CMS[37] = 91 | CMS[71] = 69 |
| CMS[4] = 97 | CMS[38] = 3 | CMS[72] = 72 |
| CMS[5] = 47 | CMS[39] = 22 | CMS[73] = 59 |
| CMS[6] = 16 | CMS[40] = 60 | CMS[74] = 25 |
| CMS[7] = 43 | CMS[41] = 20 | CMS[75] = 27 |
| CMS[8] = 84 | CMS[42] = 34 | CMS[76] = 32 |
| CMS[9] = 9 | CMS[43] = 7 | CMS[77] = 2 |
| CMS[10] = 10 | CMS[44] = 1 | CMS[78] = 68 |
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| CMS[12] = 36 | CMS[46] = 58 | CMS[80] = 19 |
| CMS[13] = 26 | CMS[47] = 5 | CMS[81] = 85 |
| CMS[14] = 31 | CMS[48] = 52 | CMS[82] = 82 |
| CMS[15] = 83 | CMS[49] = 28 | CMS[83] = 15 |
| CMS[16] = 6 | CMS[50] = 50 | CMS[84] = 8 |
| CMS[17] = 98 | CMS[51] = 24 | CMS[85] = 81 |
| CMS[18] = 79 | CMS[52] = 48 | CMS[86] = 63 |
| CMS[19] = 80 | CMS[53] = 54 | CMS[87] = 55 |
| CMS[20] = 41 | CMS[54] = 53 | CMS[88] = 92 |
| CMS[21] = 21 | CMS[55] = 87 | CMS[89] = 89 |
| CMS[22] = 39 | CMS[56] = 93 | CMS[90] = 90 |
| CMS[23] = 11 | CMS[57] = 64 | CMS[91] = 37 |
| CMS[24] = 51 | CMS[58] = 46 | CMS[92] = 88 |
| CMS[25] = 74 | CMS[59] = 73 | CMS[93] = 56 |
| CMS[26] = 13 | CMS[60] = 40 | CMS[94] = 30 |
| CMS[27] = 75 | CMS[61] = 67 | CMS[95] = 29 |
| CMS[28] = 49 | CMS[62] = 70 | CMS[96] = 65 |
| CMS[29] = 95 | CMS[63] = 86 | CMS[97] = 4 |
| CMS[30] = 94 | CMS[64] = 57 | CMS[98] = 17 |
| CMS[31] = 14 | CMS[65] = 96 | CMS[99] = 66 |
| CMS[32] = 76 | CMS[66] = 99 | |
| CMS[33] = 33 | CMS[67] = 61 | |



CMS filling example: fixed points + 3rd pairs

CMS[0] = 0	CMS[34] = 42	CMS[68] = 78
CMS[1] = 44	CMS[35] = 45	CMS[69] = 71
CMS[2] = 77	CMS[36] = 12	CMS[70] = 62
CMS[3] = 38	CMS[37] = 91	CMS[71] = 69
CMS[4] = 97	CMS[38] = 3	CMS[72] = 72
CMS[5] = 47	CMS[39] = 22	CMS[73] = 59
CMS[6] = 16	CMS[40] = 60	CMS[74] = 25
CMS[7] = 43	CMS[41] = 20	CMS[75] = 27
CMS[8] = 84	CMS[42] = 34	CMS[76] = 32
CMS[9] = 9	CMS[43] = 7	CMS[77] = 2
CMS[10] = 10	CMS[44] = 1	CMS[78] = 68
CMS[11] = 23	CMS[45] = 35	CMS[79] = 18
CMS[12] = 36	CMS[46] = 58	CMS[80] = 19
CMS[13] = 26	CMS[47] = 5	CMS[81] = 85
CMS[14] = 31	CMS[48] = 52	CMS[82] = 82
CMS[15] = 83	CMS[49] = 28	CMS[83] = 15
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CMS[17] = 98	CMS[51] = 24	CMS[85] = 81
CMS[18] = 79	CMS[52] = 48	CMS[86] = 63
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CMS[20] = 41	CMS[54] = 53	CMS[88] = 92
CMS[21] = 21	CMS[55] = 87	CMS[89] = 89
CMS[22] = 39	CMS[56] = 93	CMS[90] = 90
CMS[23] = 11	CMS[57] = 64	CMS[91] = 37
CMS[24] = 51	CMS[58] = 46	CMS[92] = 88
CMS[25] = 74	CMS[59] = 73	CMS[93] = 56
CMS[26] = 13	CMS[60] = 40	CMS[94] = 30
CMS[27] = 75	CMS[61] = 67	CMS[95] = 29
CMS[28] = 49	CMS[62] = 70	CMS[96] = 65
CMS[29] = 95	CMS[63] = 86	CMS[97] = 4
CMS[30] = 94	CMS[64] = 57	CMS[98] = 17
CMS[31] = 14	CMS[65] = 96	CMS[99] = 66
CMS[32] = 76	CMS[66] = 99	
CMS[33] = 33	CMS[67] = 61	





Computing experiments (from 07.2019)

- exp404 — 10 fixed points + 1 random pair of values + Brute Force for empty cells
- exp405 — 10 fixed points + 2 random pair of values + Brute Force for empty cells
- exp406 — 10 fixed points + 3 random pair of values + Brute Force for empty cells
- exp407 — 10 fixed points + 4 random pair of values + Brute Force for empty cells
- exp408 — 10 fixed points + 5 random pair of values + Brute Force for empty cells

Computing experiment: results

Very rare objects!

0	1	2	3	4	5	6	7	8	9
1	2	0	6	7	9	8	3	4	5
3	6	7	9	8	4	2	5	1	0
4	0	8	5	2	3	7	1	9	6
5	9	4	8	3	6	0	2	7	1
7	8	6	4	0	1	3	9	5	2
6	4	5	2	1	7	9	0	3	8
9	5	1	7	6	0	4	8	2	3
2	3	9	0	5	8	1	4	6	7
8	7	3	1	9	2	5	6	0	4

06.08.2019
1CF Loop-4 (re-find in
different WU's)

0	1	2	3	4	5	6	7	8	9
1	2	0	4	5	7	9	8	6	3
5	0	1	6	3	9	8	2	4	7
9	3	5	8	2	1	7	4	0	6
4	6	3	5	7	8	0	9	2	1
8	4	6	9	1	3	2	5	7	0
7	8	9	0	6	4	5	1	3	2
2	9	4	7	8	0	3	6	1	5
6	5	7	1	0	2	4	3	9	8
3	7	8	2	9	6	1	0	5	4

09.08.2019
1CF Once (**new!**)

- +6 additional 1 CF Onces (**new!**) and 15 re-find 1 CF Onces

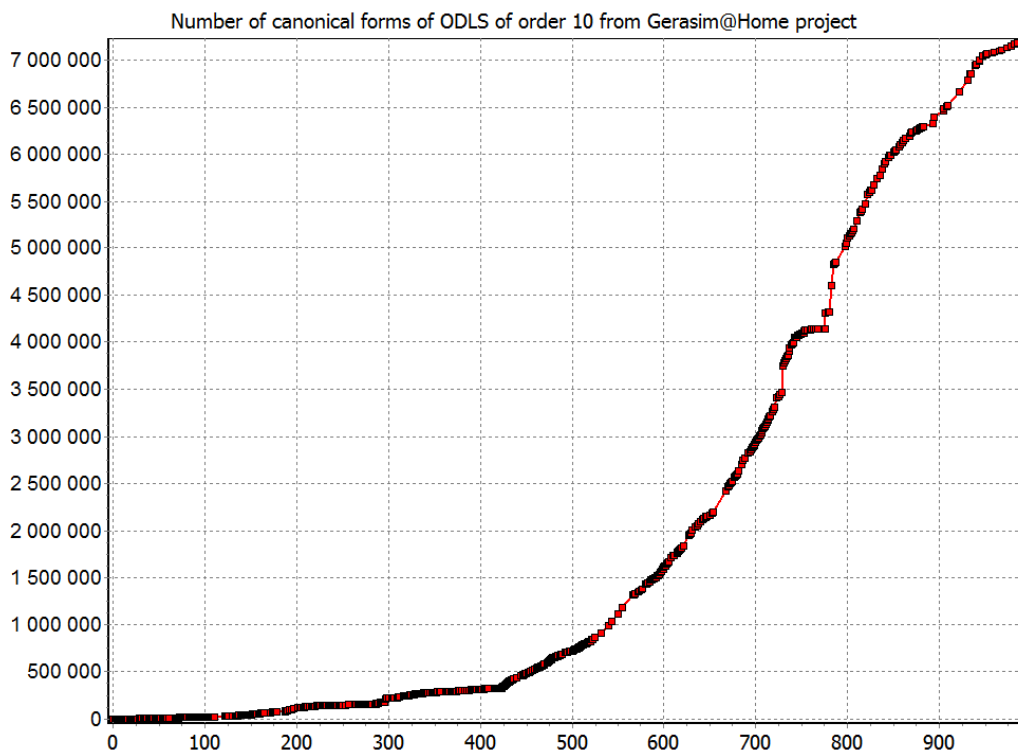


Computing experiments (from 07.2019)

- exp404 — 35 CFs / 1 none repeating / 0 new
- exp405 — 4 CFs / 4 none repeating / 3 new (schemas 44, 90, 185)
- exp406 — 4 CFs / 3 none repeating / 1 new (schema 49)
- exp407 — 7 CFs / 7 none repeating / 1 new (schema 185)
- exp408 — 6 CFs / 6 none repeating / 1 new (schema 180)

Getting ODLS CFs within Gerasim@Home project

Strategy of search: getting source square (random generator, symmetric random generator), try to get orthogonal square, add the unique CF to collection



- Brute Force with bits arithmetic (03.2017)
- DLX v1, array (04.2017)
- DLX v2, pointers (05.2017)
- SN DLS (SCFs) (08.2017)
- horizontal symmetry (10.2017)
- different canonization strategy (04.2018)
- neighborhoods of generalized symmetries (2018)





Related work

Collecting CFs and new combinatorial structures search:

- triple of MODLS (is it exist?)
- different structures?

GPU implementation of transversal, cover and ESODLS algorithms?

Enumeration problems (OEIS):

- expanding current sequences
- enumerating DLS and ODLS of special kind (string-inverse, symmetric, ...) and its CFs

Pseudo triples:

- 3 kinds of pseudo triples, only 1 was investigated in details



Thank you for your attention!

Thanks to all the volunteers who took part in the
Gerasim@home project!

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